Catalan functions and k-Schur functions

Abstract:

Li-Chung Chen and Mark Haiman studied a family of symmetric functions called Catalan (symmetric) functions which are indexed by pairs consisting of a partition contained in the staircase \((n-1, \ldots, 1, 0)\) (of which there are Catalan many) and a composition weight of length \(n\). They include the Schur functions, the Hall-Littlewood polynomials and their parabolic generalizations. They can be defined by a Demazure-operator formula, and are equal to the \(GL\)-equivariant Euler characteristics of vector bundles on the flag variety by the Borel-Weil-Bott Theorem. We have discovered various properties of Catalan functions, providing new insight on the existing theorems and conjectures inspired by the Macdonald Positivity Conjecture.

A key discovery in our work is an elegant set of ideals of roots whose associate Catalan functions are \(k\)-Schur functions, proving that graded \(k\)-Schur functions are \(GL\)-equivariant Euler characteristics of vector bundles on the flag variety, settling a conjecture of Chen-Haiman. We exposed a new shift-invariance property of the graded \(k\)-Schur functions and resolved the Schur positivity and \(k\)-branching conjectures by providing direct combinatorial formulas using strong marked tableaux. We conjectured that Catalan functions with a partition weight are \(k\)-Schur positive which strengthens the Schur positivity of the Catalan function conjecture by Chen-Haiman and resolved the conjecture with positive combinatorial formulas in cases which capture and refine a variety of problems.

This is joint work with Jonah Blasiak, Jennifer Morse, and Daniel Summers.

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