Abstract:

The coordinate ring $S = \mathbb{C}[x_{i,j}]$ of the space of $m \times n$ matrices carries an action of the group $GL_m \times GL_n$ via row and column operations on the matrix entries. If we consider any $GL_m \times GL_n$-invariant ideal $I$ in $S$, the syzygy modules $\text{Tor}_i(I, \mathbb{C})$ will carry a natural action of $GL_m \times GL_n$. By the BGG correspondence, they also carry an action of $\Lambda^\bullet(\mathbb{C}^m \otimes \mathbb{C}^n)$. It turns out that we can combine these actions together and make them modules over the general linear Lie superalgebra $\mathfrak{gl}(m \mid n)$. We will explain how this works and how it enables us to commute all Betti number of any $GL_m \times GL_n$-invariant ideal $I$. This latter part will involve combinatorics of Dyck paths.

Host: Steven Sam

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