Abstract:
In this talk, we discuss a construction of the discretization of classical field theories, within the Lagrangian and Hamiltonian frameworks, which preserve the various underlying structures inherent to the physical theories. Preservation of structure under discretization is desirable as it ensures similar behavior between the discretized field dynamics and the actual field dynamics, and often provides computational benefits such as long-term stability and reduction of numerical artifacts. We present a Discrete Lagrangian and Discrete Hamiltonian approach to structure-preserving discretization of field theories. As a motivating example, we apply these methods, in conjunction with discretization spaces from the Finite Element Exterior Calculus, to construct a discretization of classical Yang-Mills theories arising in particle physics. As a simple numerical example, we discretize electromagnetism coupled to particle-in-cell plasma dynamics. To conclude, we briefly discuss future directions for research, including group-equivariant interpolation applied to the discretization of gauge theories, connections with the discretization of quantum field theories, and methods for studying the (generally nonlinear) dynamics of the discretized fields.