Abstract:

A del Pezzo fibration is one of the natural outputs of the Minimal Model Program for threefolds. At the same time, geometry of an arbitrary del Pezzo fibration can be unsatisfying due to the presence of non-integral fibers and terminal singularities of an arbitrarily large index. In 1996, Corti developed a program of constructing ‘standard models’ of del Pezzo fibrations within a fixed birational equivalence class. Standard models enjoy a variety of desired properties, one of which is that all of their fibers are Q-Gorenstein integral del Pezzo surfaces. Corti proved the existence of standard models for del Pezzo fibrations of degree $d \geq 2$, with the case of $d = 2$ being the most difficult. The case of $d = 1$ remained a conjecture. In 1997, Kollár recast and improved the Cortis result in degree $d = 3$ using ideas from the Geometric Invariant Theory for cubic surfaces. I will present a generalization of Kollár’s approach in which we develop notions of stability for families of low degree ($d \leq 2$) del Pezzo fibrations in terms of their Hilbert points (i.e., low degree equations cutting out del Pezzos). A correct choice of stability and a bit of enumerative geometry then leads to (very good) standard models in the sense of Corti. This is a joint work with Hamid Ahmadinezhad and Igor Krylov.