Etale cohomology of algebraic varieties over the maximal cyclotomic extension of a global field

Abstract:

Let $k$ be a global field, that is, a number field of finite degree over $\mathbb{Q}$ or the function field of a smooth projective curve $C$ over a finite field $F$. Let $X$ be a smooth projective variety over $k$, and let $K$ be the maximal cyclotomic extension of $k$, obtained by adjoining all roots of unity. If $X$ is an abelian variety, a famous theorem, due to Ribet in the number field case and Lang-Neron in the function field case when $X$ has trace zero over the constant subfield of $K$, asserts that the torsion subgroup of the Mordell-Weil group of $X$ over $K$ is finite. Denoting by $k^{sep}$ a separable closure of $k$, this result is equivalent to finiteness of the fixed part by $G = \text{Gal}(k^{sep}/K)$ of the etale cohomology group $H^1(X_{k^{sep}}, \mathbb{Q}/\mathbb{Z})$, where we ignore the $p$-part in positive characteristic $p$. In a recent paper, Roessler-Szamuely generalize this result to all odd cohomology groups. The trace zero assumption in the function field case is replaced by a "large variation" assumption on the characteristic polynomials of Frobenius acting on the cohomology of the fibres of a morphism $f : \mathcal{X} \to C$ from a smooth projective variety $\mathcal{X}$ over a finite field to $C$ with generic fibre $X$. In this talk, I will discuss the case of even degree, proving some positive results in the number field case and negative results in the function field case.

Special Note:
There will be a preparatory talk for graduate students and postdocs 3:20-3:50pm in the seminar room.

Host: Cristian Popescu

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