Math 288 - Probability Seminar

Prof. Larry Goldstein

Dickman Approximation in Quickselect sorting and Probabilistic Number Theory

Abstract:

The generalized Dickman distribution $D_{\theta}$ with parameter $\theta > 0$ is the unique solution to the distributional equality $W =_d W^*$, where

$$W^* =_d U^{1/\theta}(W + 1),$$

with $W$ non-negative with probability one, $U \sim U[0, 1]$ independent of $W$, and $=_d$ denoting equality in distribution. Members of this family appear in the study of algorithms, number theory, stochastic geometry, and perpetuities.

The Wasserstein distance $d(\cdot, \cdot)$ between such a $W$ with finite mean, and $D \sim D_{\theta}$ obeys

$$d(W, D) \leq (1 + \theta)d(W^*, W).$$

The specialization of this bound to the case $\theta = 1$ and coupling constructions yield for $n \geq 1$ that

$$d_1(W_n, D) \leq \frac{10 \log(n/2) + 10}{n} \quad \text{where} \quad W_n = \frac{1}{n}C_n - 1,$$

and $C_n$ is the number of comparisons made by the Quickselect algorithm to find the smallest element of a list of $n$ distinct numbers.

Joint with Bhattacharjee, using Stein’s method, bounds for Wasserstein type distances can also be computed between $D_{\theta}$ and weighted sums arising in probabilistic number theory of the form

$$S_n = \frac{1}{\log(p_n)} \sum_{k=1}^{n} X_k \log(p_k)$$

where $(p_k)_{k \geq 1}$ is an enumeration of the prime numbers in increasing order and $X_k$ is, for instance, Geometric with parameter $1 - 1/p_k$.

Host: Todd Kemp

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