Comparing gauge theoretic invariants of homology $S_1 \times S_3$

Abstract:
Since the ground breaking work of Donaldson in the 1980s, topologists has achieved huge success in using gauge theory to study smooth 4-manifolds with nonzero second homology. The case of 4-manifolds with trivial second homology is relatively less known. In particular, when the 4-manifold have the same homology as $S_1 \times S_3$, there are several gauge theoretic invariants. The first one is the Casson-Seiberg-Witten invariant $LSW(X)$ defined by Mrowka-Ruberman-Saveliev; the second one is the Fruta-Ohta invariant $LFO(X)$. It is conjecture that these two invariants are equal to each other (This is an analogue of Witten’s conjecture relating Donaldson and Seiberg-Witten invariants.) In this talk, I will recall the definition of these two invariants, give some applications of them (including a new obstruction for metric with positive scalar curvature), and sketch a proof of this conjecture for finite-order mapping tori. This is based on a joint work with Danny Ruberman and Nikolai Saveliev.