Abstract:

Let $(X, T^{1,0}X)$ be a compact CR manifold and $(L, h)$ be a Hermitian CR line bundle over $X$. When $X$ is Levi-flat and $L$ is positive, Ohsawa and Sibony constructed for every $\kappa \in \mathbb{N}$ a CR projective embedding of $C^\kappa$-smooth of the Levi-flat CR manifold. Adachi constructed a counterexample to show that the $C^k$-smooth can not be generalized to $C^\infty$-smooth. The difficulty comes from the fact that the Kohn Laplacian is not hypoelliptic on Levi flat manifolds.

In this talk, we will consider CR manifold $X$ with a transversal CR $G$-action where $G$ is a compact Lie group and $G$ can be lifted to a CR line bundle $L$ over $X$. The talk will be divided into two parts. In the first part, we will talk about the Morse inequalities for the Fourier components of Kohn-Rossi cohomology on CR manifolds with transversal CR $S^1$-action. By studying the partial Szegő kernel on $(0, q)$-forms with values in $L^k$ we obtain the Morse inequalities on $X$ without any Levi form assumption. In the second part, when the CR line bundle $L$ is positive, the Kodaira embedding theorems for CR manifold with $G$-action when $G$ is $S^1$, Torus and $\mathbb{R}$ will be presented. As an application, this will generalize Ohsawa and Sibony’s result to $C^\infty$-smooth in our setting.