Abstract:

Suppose that an infinite subset \( A \) of the natural numbers is partitioned into finitely many subsets. A property of \( A \) that is always inherited by at least one of the elements of the partition is known as a partition regular property. Suppose we have a method of measuring the size of an infinite subset of the natural numbers. A property \( P \) is said to be a density property if every infinite subset of the natural numbers which is large enough, according to our yardstick, satisfies \( P \). In the qualitative sense, partition regular properties guarantee that order is conserved and density properties guarantee that order is always achieved at a threshold size. We will provide some interesting examples and applications of partition regular properties, density properties, and related ideas.