Math 295 - Mathematics Colloquium

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Growth rates of invariant random subgroups of Lie groups and hyperbolic groups

Abstract:

Invariant random subgroups (IRS) are conjugacy invariant probability measures on the space of subgroups of a given locally compact group G. They arise naturally as point stabilizers of probability measure preserving actions. Invariant random subgroups can be regarded as a generalization both of normal subgroups and of lattices in topological groups. As such, it is interesting to extend results from the theories of normal subgroups and of lattices to the IRS setting. Stuck-Zimmer proved that for higher rank simple Lie groups, any nontrivial IRS comes from a lattice. In rank 1 however the situation is far more complex. Indeed, the space of invariant random subgroups of $SL_2\mathbb{R}$ contains all moduli spaces of Riemann surfaces, and can be used to obtain an interesting compactification thereof related to the Deligne-Mumford compactification.

Nevertheless, jointly with Arie Levit, we prove a different type of rigidity result valid in the rank 1 setting. We show that the critical exponent (exponential growth rate) of an infinite IRS in an isometry group of a Gromov hyperbolic space (such as a rank 1 Lie group, or a hyperbolic group) is almost surely greater than half the Hausdorff dimension of the boundary. This can be reinterpreted by saying that for any probability measure preserving action of such a group, stabilizers are almost surely either trivial or “very big”. This generalizes an analogous result of Matsuzaki-Yabuki-Jaerisch for normal subgroups. As a corollary, we obtain that if $\Gamma$ is a typical subgroup and $X$ a rank 1 symmetric space then $\lambda_0(X/\Gamma) < \lambda_0(X)$ where $\lambda_0$ is the smallest eigenvalue of the Laplacian. The proof uses ergodic theorems for actions of hyperbolic groups.

Hosts: Adrian Ioana and Alireza Salehi Golsefidy

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