Abstract:
Minimal surfaces (critical points of the area functional) have a rich and successful history in the study of the interaction between geometry and topology that goes back to the 1960s. In practice, the presence and properties of minimal surfaces inside a Riemannian manifold profoundly influences the ambient geometry. In this talk, we will discuss how one can use the Allen–Cahn equation to guarantee the existence of a rich class of geometrically and topologically distinct minimal surfaces inside a generic Riemannian 3-manifold. As a byproduct, one obtains a pure PDE resolution of a number of previously unapproachable questions in minimal surface theory, which parallels recent simultaneous advances that instead use geometric measure theory.