Abstract:

In this talk I will survey recent work with Kleiner in which we verify two topological conjectures using Ricci flow. First, we classify the homotopy type of every 3-dimensional spherical space form. This proves the Generalized Smale Conjecture and gives an alternative proof of the Smale Conjecture, which was originally due to Hatcher. Second, we show that the space of metrics with positive scalar curvature on every 3-manifold is either contractible or empty. This completes work initiated by Marques. Our proof is based on a new uniqueness theorem for singular Ricci flows, which I have previously obtained with Kleiner. Singular Ricci flows were inspired by Perelman’s proof of the Poincaré and Geometrization Conjectures, which relied on a flow in which singularities were removed by a certain surgery construction. Since this surgery construction depended on various auxiliary parameters, the resulting flow was not uniquely determined by its initial data. Perelman therefore conjectured that there must be a canonical, weak Ricci flow that automatically “flows through its singularities” at an infinitesimal scale. Our work on the uniqueness of singular Ricci flows gives an affirmative answer to Perelman’s conjecture and allows the study of continuous families of singular Ricci flows leading to the topological applications mentioned above.