Markov chain mixing speedup on the cycle by adding a single random edge and dropping reversibility

Abstract:
For any reversible Markov chain on the cycle with uniform stationary distribution, it is well known that the mixing time is $\Omega(n^2)$. We are interested in better understanding the improvement possibilities by relaxing the framework a bit. Two natural extensions are combined, first by dropping the technical condition of reversibility, second by allowing more edges as it is also motivated by certain random graph models. However, for the latter, we are very conservative: we already stop at one extra edge. Interestingly, a non-trivial speedup already emerges, the mixing time can drop to $\Theta(n^{3/2})$, provided that the added edge is appropriate in some sense.