Abstract:
Option contracts are a type of financial derivative that allow investors to hedge risk and speculate on the variation of an asset’s future market price. In short, an option has a particular payout that is based on the market price for an asset on a given date in the future. In 1973, Black and Scholes proposed a valuation model for options that essentially estimates the tail risk of the asset price under the assumption that the price will fluctuate according to geometric Brownian motion. More recently, DeMarzo et al., among others, have proposed more robust valuation schemes, where we can even assume an adversary chooses the price fluctuations. This framework can be considered as a sequential two-player zero-sum game between the investor and Nature. We analyze the value of this game in the limit, where the investor can trade at smaller and smaller time intervals. Under weak assumptions on the actions of Nature (an adversary), we show that the minimax option price asymptotically approaches exactly the Black-Scholes valuation. The key piece of our analysis is showing that Nature’s minimax optimal dual strategy converges to geometric Brownian motion in the limit.