Algebraic independence of zeta values of function fields of elliptic curves

Abstract:
Classically, the transcendence (and even the irrationality) of odd zeta values is widely conjectured, but yet unproven. However, for zeta values defined over the rational function field, Jing Yu succeeded in proving their transcendence in 1991, and many other transcendence results (including algebraic independence) followed in the intervening years. In this work (joint with T. Ngo Dac), we prove the algebraic independence of zeta values defined over the function field of an elliptic curve. The main technique we use is to construct a Tannakian category of t-motives whose associated periods contain these zeta values - thus we may exploit the existence of a motivic Galois group to study the transcendence degree. We also discuss the difficulties and pathway to proving algebraic independence for zeta values of function fields of arbitrary curves.