Abstract:

The Ricci flow is a natural evolution equation for Riemannian metrics on a manifold. From a PDE perspective, the Ricci flow is a system of linear parabolic equations, which can be viewed as the heat equation analogue of the Einstein equations in general relativity. The central problem is to understand singularity formation. In other words, what does the geometry look like at points where the curvature is large? In his spectacular 2002 breakthrough, Perelman showed that, for a solution to the Ricci flow in dimension 3, the high curvature regions are modeled on so-called ancient \( \kappa \)-solutions: By definition, these are solutions to the Ricci flow which are defined for \( t \in (-\infty, T] \) and satisfy a noncollapsing condition. Moreover, Perelman achieved a qualitative understanding of ancient \( \kappa \)-solutions in dimension 3; this is sufficient for topological conclusions.

In this lecture, I will discuss recent work which gives a complete classification of all ancient \( \kappa \)-solutions in dimension 3. It turns out that the only noncompact examples are shrinking cylinders (or quotients thereof), and the rotationally symmetric Bryant soliton, thereby confirming a conjecture of Perelman. Moreover, I will mention joint work with Panagiota Daskalopoulos and Natasa Sesum, which shows that the only compact examples are shrinking spheres and the rotationally symmetric Perelman ovals (or quotients thereof).