Abstract:
Kronheimer-Mrowka recently proved that the Dehn twist along a 3-sphere in the neck of $K3\#K3$ is not smoothly isotopic to the identity. This provides a new example of self-diffeomorphisms on 4-manifolds that are isotopic to the identity in the topological category but not smoothly so. (The first such examples were given by Ruberman.)

In this talk, we study the Bauer-Furuta invariant as an element in the Pin(2)-equivariant stable homotopy group of spheres. We use it to show that this Dehn twist is not smoothly isotopic to the identity even after a single stabilization (connected summing with the identity map on $S2 \times S2$). This gives the first example of exotic phenomena on simply-connected smooth 4-manifolds that do not disappear after a single stabilization. In particular, it implies that one stabilization is not enough in the diffeomorphism isotopy problem for 4-manifolds. It gives an interesting comparison with Auckly-Kim-Melvin-Ruberman-Schwartz’s theorem that one stabilization is enough in the surface isotopy problem.