Abstract:

Numerical algebraic geometry studies methods to approach problems in algebraic geometry numerically. Especially, finding roots of systems of equations using theory in algebraic geometry involves symbolic algorithm which requires expensive computations. However, numerical techniques often provides faster methods to tackle these problems. We establish numerical techniques to approximate roots of systems of equations and ways to certify its correctness.

As techniques for approximating roots of systems of equations, homotopy continuation method will be introduced. Since numerical approaches rely on heuristic method, we study how to certify numerical roots of systems of equations. Krawczyk method from interval arithmetic and Smale’s alpha theory will be used as main paradigms for certification. Furthermore, as an approach for multiple roots, we establish the local separation bound of a multiple root. For a regular quadratic multiple zero, we give their local separation bound and study how to certify an approximation of such multiple roots.