Abstract:
Let $F$ be a finite subset of $\mathbb{Z}^d$. We say that $F$ is a translational tile of $\mathbb{Z}^d$ if it is possible to cover $\mathbb{Z}^d$ by translates of $F$ without any overlaps. The periodic tiling conjecture, which is perhaps the most well-known conjecture in the area, suggests that any translational tile admits at least one periodic tiling. In the talk, we will motivate and discuss the study of this conjecture. We will also present some new results, joint with Terence Tao, on the structure of translational tilings in lattices and introduce some applications.