Abstract:
We will begin by discussing the multisymplectic structure associated to Hamiltonian PDEs as a generalization of the symplectic structure associated to Hamilton’s equations in classical mechanics. Subsequently, we will turn to the question of how to computationally model such Hamiltonian PDEs while preserving the multisymplectic structure at the discrete level. This will lead us to the notion of a multisymplectic integrator. The analogue of these integrators in Hamiltonian mechanics are known as symplectic integrators, which are extremely well-studied and have proven to provide extremely robust and physically faithful simulations of mechanical systems. Multisymplectic integration, on the other hand, is still in its relative infancy.

After establishing the necessary background, we will introduce our construction of variational integrators for Hamiltonian PDEs which automatically yield multisymplectic integrators. This construction gives a systematic framework for constructing such multisymplectic integrators, based on the notion of a Type II generating functional. As an application of our framework, we will derive the class of multisymplectic partitioned Runge–Kutta methods and provide a numerical example with the family of sine–Gordon soliton solutions. This is joint work with Prof. Melvin Leok.