Math 109 Proof Checklist

The purpose of this document is to give you a list of items you can check after coming up with solutions to the homework problems but *before* you write up the final draft to turn in.

- 1. Write the problem to be solved at the top of the page. Then on a new line, write the word, "Proof." Start your proof on another new line.
- 2. Don't write anything else besides the problem to be solved above the line that says "Proof." If you write extra information, miscellaneous definitions, or subsidiary facts above the word "Proof" it becomes unclear which of all the facts you listed are being proved. Incorporate definitions and related facts into the body of the proof at the points where they are required by the proof.
- 3. Don't write what is to be shown as the first line of your proof. Any time you write any statement in the body of the proof, it is assumed that you have already established the truth of that statement. For this reason, the statement that is to be proved should not appear again until the last line of the proof.
- 4. Indicate the proof technique you are using. If you don't, the first steps of your proof might be confusing. For example, if you are using proof by contradiction, but you don't state this, it will seem odd that you are assuming the opposite of what you're trying to prove.
- 5. On the other hand, you should give some idea in the body of the proof where you are headed so that it is not just a loose collection of facts that you may or may not tie together by the end of the proof. In the event you need to clarify where you are headed, make sure you use a phrase such as, "We need to prove that...," or, "In order to establish the next result, we need to show...."
- 6. Your formal write up needs to be "linear," meaning that each sentence must logically follow from the preceding ones. This will not always be the exact same form and order in which the steps of the proof occurred to you while working on the problem. While it's acceptable and beneficial to attack a problem from different angles, the end result must never state a fact not already established earlier in the proof.

7. Avoid backward reasoning. Backward reasoning occurs when you need a particular fact in the course of a proof and so you state that fact, followed immediately by, "...and this is true because...." The problem here is that the statement of the fact precedes its proof. Remember that any time you write any statement in the body of the proof, it is assumed that you have already established the truth of that statement.

Backward reasoning is a great way to investigate the solution of a problem. You might think to yourself, "Let's see. I know that the goal is to arrive at this statement. In order to show this statement, I will need to show this other statement. Of course, to show that, I'll need to establish this other result," and so forth. But this should not be the format of the final product you turn in.

Another type of backward reasoning involves equations and inequalities. To prove an equation, for example, it is not correct to write the equation and then start performing algebra on both sides of the equation until you work your way down to another equation you believe to be true. This is the wrong way around. To fix the problem, start with the equation you already know to be true and see if you can manipulate both sides of this equation until you arrive at the equation you wish to prove. (Oftentimes, though, a more efficient way to prove an equation or an inequality is to start with the expression on the left hand side and do algebraic manipulations until you arrive at the expression on the right hand side. This will help you avoid backward reasoning.)

- 8. Avoid phrases like, "It is clear that...," and, "It is obvious that...." Yes, I know the book uses these phrases sometimes, as do countless other math texts. It is a bad habit to get into. If you want to state something that truly is obvious like, "6 is an even number," then just state it. Otherwise, provide a proof anyway, just to be on the safe side. If it is so "obvious" what you are stating, it shouldn't require a very long proof anyway.
- 9. Don't substitute logical symbols for clear English. Logical symbols have their place in the study of logic, and sometimes it might be helpful to your investigation of a solution to try to write things out in symbols to make sure you are proving the correct facts. For example, if the statement to be proved is that some implication $P \to Q$ is not true,

then writing out the negation in symbols helps remind you that you need to prove $P \wedge \neg Q$, or in other words, you are supposed to show both the statement P is true and that the statement Q is false.

Nevertheless, when it comes time to write up your homework to turn in, state everything with English words.

- 10. Use correct spelling and grammar. Your English or history professors will definitely lower your grade for errors of spelling and grammar, and it's just as important in math that your reasoning be clear and concise.
- 11. Use paragraphs. It's difficult to read and decipher a proof that goes on for a whole page without a break. Divide up the main chunks of your proof and put each in a new paragraph. The initial sentence of each paragraph can be something like, "The next step is to prove that...."
- 12. Use good handwriting. Some students even opt to type up their assignments, although this is not required. It's also a bit more difficult to type up your proofs since the average word processor has some difficulty producing correctly formatted mathematical symbols. Nevertheless, since most of your proof should consist of English sentences, this isn't such an impediment.
- 13. Follow the guidelines posted on the website for formatting your paper. In particular, double check before turning in your assignment that your name and the assignment number appear on every page you turn in since this is the error that has cost people the most points up to this point.