Math 10A Final Review Outline

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Chapter 1: Library of Functions

Section 1.1: Functions and Change

- Know the formula for a linear function: $y = mx + b$, where m is the slope and b is y-intercept.
- Given two points, know how to compute m, the slope.
- Know how to tell the difference between different lines (look at slopes/y-intercepts)
- Given a function, interpret its meaning.
  - If $P(x)$ is the price of x units, what is the meaning of $P^{-1}(200)$?

Section 1.2: Exponential Functions

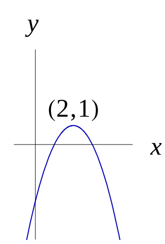
- Know the general exponential function: $P = P_0 a^t$, where P_0 is the initial quantity, and a is the factor by which P changes when t increases by 1.
- Given two points on an exponential curve, know how to find the equation
  - If $f(1) = 12$, $f(3) = 108$, find a formula for $f(t) = Q_0 a^t$.
  - The size of a bacteria colony grows exponentially as a function of time. If the size of the bacteria colony doubles every 3 hrs, how long will it take to triple?
  - The fraction of a lake's surface covered by algae was initially 0.42 and was halved each year since the passage of anti-pollution laws. How long after the passage of the law was only 0.07 of the lake's surface covered with algae?
  - In 1924, Granny invested $75 (the contents of her purse) at a fixed annual interest rate. In 1964, her investment was worth $528. How much is her investment worth today (2008)?
  - The number of people who have heard a rumor is 10 at 6:00am and from that point doubles every 20 minutes. When have 100 people heard the rumor?

Section 1.3: New Functions From Old

Know the different types of shifts and what they do to a graph

1. $f(x-h)+k$ translates h units to the right, k units vertically
2. $-f(x)$ reflects across x-axis
3. $f(-x)$ reflects across y-axis
4. $cf(x)$ dilates by factor of c vertically
5. $f(cx)$ dilates by factor of $\frac{1}{c}$ horizontally

- Know some types of example questions using the above shifts
  - How are the graphs of $y=(x+2)^2$ and $y=x^2$ related?
  - What would I need to do to a graph to reflect it about the y-axis and shift it up 3 units?
  - The graph below was made from $y=x^2$ by reflecting it about the x-axis, shifting it to the right by 2, and up by 1. Find its equation.



- Know what it means for a function to be even or odd
- Know how the graph of $f(x)$ and $f^{-1}(x)$ are related (reflection about line $y=x$)
- Know how to find the domain of a function
  - Find the domain of $f(x)=\frac{x^2+1}{x^2-4}$
- Know the relationship between the domain/range of $f(x)$ and $f^{-1}(x)$. That is, the domain (range) of $f(x)$ is the range (domain) of $f^{-1}(x)$
  - Find the domain of $f^{-1}(x)$ by considering the range of $f(x)=\sqrt{x-1}$
- Know what $f(g(x))$ means, (that is, plug $g(x)$ into $f(x)$) and how to do this.
  - If $f(x)=\frac{x}{x-1}$, $g(x)=3x+1$, what is $f(g(x))$? $f(g(3))$? $f(f(2))$? $f(g^{-1}(1))$?

Section 1.4: Logarithmic Functions

- Know how to use logarithms to solve exponential problems
- Know the properties of logarithms and be careful of the false properties:

True	False
$\ln AB = \ln A + \ln B$	$\ln A + B = \ln A + \ln B$
$\ln A/B = \ln A - \ln B$	$lnA/lnB = lnA - lnB$
$\ln A^p = p \ln A$	

Section 1.5: Trigonometric Functions

- Know the graphs of $y = A\sin(Bt) + C$ and $y = A\cos(Bt) + C$
- Know how B and the period are related (Period $= 2\pi \times \frac{1}{B}$)
- Know how to find the amplitude, $|A|$ ($|A| = \frac{max - min}{2}$)
- Know how to find the vertical shift, C ($C = max - |A|$)
- Know the basic values of $\sin(t)$ and $\cos(t)$. What is $\sin(0)$? What is $\cos(0)$? etc.
- Know how to tell the differences between various Sine/Cosine graphs.

Section 1.6: Powers, Polynomials, and Rational Functions

- Know what it means for $f(x)$ to be a rational function. ($f(x) = \frac{p(x)}{q(x)}$).
- Know how to find horizontal asymptotes of rational functions
  - Find the horizontal asymptote of $y = \frac{4x^2+1}{x^3-1}$ and of $y = \frac{10}{5+e^{-t}}$.
- Know where a rational function is undefined (where it has vertical asymptotes).
  - Where is the function $f(x) = \frac{x^2+4x+3}{x+1}$ undefined?
- Know how to find asymptotes and domain/range of a function
  - Find the domain of $f(x) = \frac{x+1}{2x+1}$ undefined. Find the horizontal/vertical asymptotes of $f(x)$. Find $f^{-1}(x)$ and its domain.

Section 1.7: Introduction to Continuity

- Know what it means for a function to be continuous (the graph has no breaks/holes).
- Know how to make a rational function continuous at every point
  - Find k such that $f(x) = \begin{cases} \frac{x^2+4x+3}{x+1} & x \neq -1, \\ k & x = -1 \end{cases}$ is continuous on any interval.

Section 1.8: Limits

- Know when a limit exists or does not exist and how to evaluate limits algebraically
  - Evaluate $\lim_{h \to 0} \dfrac{\frac{4}{1+h} - 4}{h}$ algebraically.

Chapter 2: Key Concept: The Derivative

Section 2.1: How Do We Measure Speed?

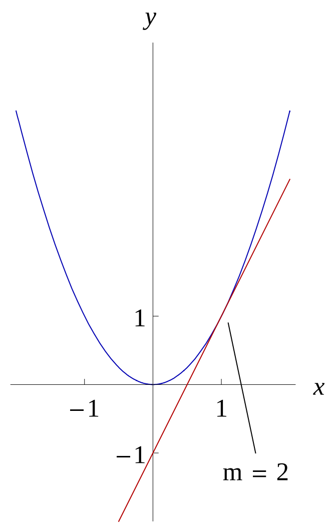
- Know how to use average rates of change (i.e. slope) to estimate the derivative at a point (Think the difference quotient, which is just another equation for slope)
  - If $g(x) = x^2 + 1$, estimate $g'(2)$ using average rates of change. (Hint: Use two points near 2, for example 2 and 2.01.).
  - Suppose a particle's distance from the origin is given by $\frac{1}{(3+t)}$. Find the average velocity of the particle between $t = 0$ and $t = 2$ seconds. What is the instantaneous velocity at $t = 2$ seconds?

Section 2.2: The Derivative at a Point

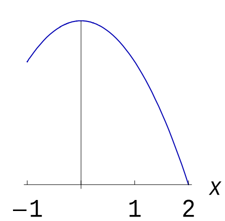
- Know the definition of the derivative (at a point). The formula is $f'(a) = \lim_{h \to 0} \dfrac{f(a+h) - f(a)}{h} = \lim_{x \to a} \dfrac{f(x) - f(a)}{x - a}$

  - If $f(x) = (x+1)^2 - 2$, find $f'(0)$ using the definition of the derivative.

- Know that the derivative at a point is the slope of the tangent line to the curve at that point.
  - The slope, m, of the tangent line of $y = x^2$ at $x = 1$ is 2. Visually, we have the picture below.



- Know how to order various slopes
  - List quantities in increasing order: $f'(0)$, $f'(1)$, $f'(2)$, $\dfrac{f(1)-f(0)}{1-0}$, $\dfrac{f(2)-f(1)}{2-1}$ in the picture below.



Section 2.3: The Derivative Function

- Know the definition of the derivative (as a function). The formula is $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

- Know what the derivative tells us graphically:
  - ⋆ If $f'(x) > 0$ on an interval, then $f(x)$ is *increasing* over that interval.
  - ⋆ If $f'(x) < 0$ on an interval, then $f(x)$ is *decreasing* over that interval.
- Note: Just because $f(x)$ is increasing does not mean that $f'(x)$ is increasing. It only means that $f'(x) > 0$. It is easy to confuse the two.
- Know the power rule for taking derivatives: $\frac{d}{dx}(ax^n) = anx^{n-1}$, where a is a constant.
- As a special case of the above, if $f(x) = c$ (a constant), then $f'(x) = 0$ for all x.
- Know that $f(x)$ may have a local max/min wherever $f'(x) = 0$.[1]

Section 2.4: Interpretation of the Derivative

- Know the units of the derivative given a function and what it means in practical terms.
  - To save energy, people are advised to turn their heaters off at night. The temperature T of the house is a function of the number of hours, x, after the heat has been turned off, so $T = f(x)$.
    1. Explain the meaning of $f(5) = 25$.
    2. Explain the meaning of $f'(5) = -3$.
    3. Explain why $f'(x)$ should be negative.
    4. What are the units of $f'(5) = -3$?
  - Investing \$1000 at an annual balance of $r\%$, compounded continuously, for 10 years gives you a balance \$$B$, where $B = g(r)$.
    1. What is the meaning of $g(6) = 1822$?
    2. What is the meaning of $g'(6) = 182$?
    3. What are the units of $g'(6) = 182$?
  - The time T in seconds that a rat takes to complete a maze is a function of the amount C of cheese (in grams) placed at the end of the maze, so $T = f(C)$.
    1. Interpret the statement $f(15) = 25$.
    2. Interpret the statement $f'(15) = -2$.

[1]The classic counterexample is $f(x) = x^3$. We have that $f'(x) = 0$ at $x = 0$, but that point is not a max or a min. To determine if a point is a local max/min, look at $f'(x)$ to the left and right of the critical point. If the derivative changes from positive to negative you have a local min. If the derivative changes from negative to positive, you have a local max. Otherwise, you do not have a local max or a local min.

Section 2.5: The Second Derivative

- Know what the second derivative tells us about $f(x)$ and $f'(x)$.
  - ⋆ If $f''(x) > 0$, then $f'(x)$ is increasing and $f(x)$ is concave up.
  - ⋆ If $f''(x) < 0$, then $f'(x)$ is decreasing and $f(x)$ is concave down.
- The following table should help to keep the relationships straight.

The graph of $f(x)$	Sign of $f'(x)$	Sign of $f''(x)$
Increasing, concave up	Positive	Positive
Increasing, concave down	Positive	Negative
Decreasing, concave up	Negative	Positive
Decreasing, concave down	Negative	Negative

- Know that $f(x)$ may change concavity wherever $f''(x) = 0$.[2]
- Note: $f''(x)$ is the derivative of $f'(x)$. So, the same properties that we discussed for the first derivative apply to these two functions.
- Given the graph of $f'(x)$, know where $f(x)$ is increasing/decreasing, concave up/down
  - Let $f(x) = \dfrac{1}{x^2+9}$. Compute $f''(x)$. On what interval(s) is $f(x)$ concave down?
  - Let $f(x) = x^2e^x$. Where is $f(x)$ increasing? Where is $f(x)$ concave down?
  - Find the values of a and b such that $(1, 6)$ is a point of inflection for the curve $y = x^3 + ax^2 + bx + 1$.

Section 2.6: Differentiability

- Know the relationship between derivatives and continuity.
- If a function is differentiable, then it is automatically continuous. The converse is *not necessarily* true!
  - The function $y = |x|$ is continuous, but not differentiable at $x = 0$.
- So, that means that if $f(x)$ is not continuous at $x = a$, then $f(x)$ is not differentiable at $x = a$.

[2] The classic counterexample is $f(x) = x^4$. We have that $f''(x) = 0$ at $x = 0$, but that point is not a place where the concavity changes. To determine if a point is a point of inflection, look at $f''(x)$ to the left and right of the critical point. If the second derivative changes signs, you have a point of inflection. Otherwise, you do not have a point of inflection.

- $f(x)$ is not differentiable at any sharp point, i.e. where we can fit more than one tangent line
- $f(x)$ is not differentiable wherever the function has a vertical slope

Chapter 3: Short-Cuts to Differentiation

Section 3.1: Powers and Polynomials

- If you have a constant times a function that you are taking the derivative of, you can factor out the constant.
  - $\frac{d}{dx}(3x^2) = 3\frac{d}{dx}(x^2) = 3(2x) = 6x$.
- If you are taking the derivative of the sum/difference of two functions, you can take the derivative of each piece
  - $\frac{d}{dx}(x^3 - x) = \frac{d}{dx}(x^3) - \frac{d}{dx}(x) = (3x^2) - (1) = 3x^2 - 1$.
- A special case of the power rule is $\sqrt{x}$.
  - $\dfrac{d}{dx}(\sqrt{x}) = \dfrac{d}{dx}(x^{1/2}) = \dfrac{1}{2}x^{-1/2} = \dfrac{1}{2\sqrt{x}}$.
- Know how to find the tangent line to a graph at a point.
  - Find the equation of the tangent line of $y = x^2$ at the point (1,1).

Section 3.2: The Exponential Function

- Know that $\dfrac{d}{dx}(a^x) = (\ln a)a^x$.
  - A special case is $a = e$. $\dfrac{d}{dx}(e^x) = (\ln e)e^x = e^x$.

Section 3.3: The Product and Quotient Rules

- Know the product and quotient rules and how to apply them to solve problems.

  ⋆ Product Rule: $\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$.

  ⋆ Quotient Rule: $\frac{d}{dx}(\frac{f(x)}{g(x)}) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$.

  – Find the derivative of (i) x^2e^x (ii) $\frac{e^x}{\sin(x)}$ (iii) $\frac{4x\cos(x)}{e^{-x}+1}$.

Section 3.4: The Chain Rule

- Know the chain rule and how to apply it to solve problems.

  ⋆ Chain Rule: $\frac{d}{dx}(f(g(x))) = f'(g(x))g'(x)$.

  – Find the derivative of (i) $\frac{x}{\sqrt{x^2+5}}$ (ii) $\frac{e^{\cos(x)}}{x^3+x}$ (iii) $\sqrt{x}(3x^2 - \sqrt{x})$.

- Know how to find the equation of a tangent line using any of the above rules.

  – Find the equation of the tangent line of $\frac{x}{\sqrt{x^2+5}}$, at $x = 2$.

  – Find the equation of the tangent line of $y = \sin(\sin(x))$ at the point $(2\pi, 0)$.

- Given a table of data, know how to compute various derivatives.

  – Suppose $f(1) = 2$, $f'(1) = 3$, $f(2) = 0$, $f'(2) = 1$, $g(1) = -1$, $g'(1) = 4$, $g(2) = 1$, $g'(2) = 5$. Compute

    (i) $h'(1)$ if $h(x) = f(x)g(x)$,
    (ii) $h'(2)$ if $h(x) = f(g(x))$,
    (iii) $h'(1)$ if $h(x) = [f(x)]^2$,
    (iv) $h'(1)$ if $h(x) = g(f(x))$.

Section 3.5: The Trigonometric Functions

- Know the relationship between sine and cosine. The following table summarizes all of this nicely.

$$\frac{d}{dx} \quad \begin{array}{c} \sin(x) \\ \searrow \quad \cos(x) \\ \searrow \quad -\sin(x) \\ \searrow \quad -\cos(x) \\ \searrow \quad \sin(x) \end{array}$$

Also, $\frac{d}{dx}(\tan(x)) = \frac{1}{\cos^2(x)} = \sec^2(x)$

  - What is the derivative of $\sin(e^x)\cos(2x)$?
  - What the slope of the tangent line of $f(x) = \tan(x - \frac{\pi}{4})$ at $(\frac{\pi}{2}, 1)$?

Section 3.6: The Chain Rule and Inverse Functions

- Know the following derivatives (these are good to put on a cheat sheet...)

$\frac{d}{dx}(\ln(g(x))) = \frac{g'(x)}{g(x)}$ [As a special case, $\frac{d}{dx}(\ln(x)) = \frac{1}{x}$]

$\frac{d}{dx}(a^{g(x)}) = (\ln a)a^{g(x)}g'(x)$ [As a special case, $\frac{d}{dx}(a^x) = (\ln a)a^x$]

$\frac{d}{dx}(\arctan g(x)) = \frac{g'(x)}{1 + g(x)^2}$ [As a special case, $\frac{d}{dx}(\arctan x) = \frac{1}{1 + x^2}$]

$\frac{d}{dx}(\arcsin g(x)) = \frac{g'(x)}{\sqrt{1 - g(x)^2}}$ [As a special case, $\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1 - x^2}}$]

  - What is the derivative of (i) $f(x) = \arcsin(x^2)$ and (ii) $g(x) = \ln e^{3\sin x}$?

Section 3.7: Implicit Functions

- Know how to find the derivative of a curve using implicit differentiation.
  - Find the derivative, $\frac{dy}{dx}$, to the curve $\ln xy = 3x$.
  - Consider the curve defined by the equation $xy^2 = \cos(x) + \sin(y)$. Find $\frac{dy}{dx}$ and the tangent line approximation at the point $(\frac{\pi}{2}, 0)$.

Section 3.9: Linear Approximations and the Derivative

- Know the formula for the tangent line approximation to $f(x)$
  - ⋆ $f(x) \approx f(a) + f'(a)(x - a)$.
    - Note: This is just the point-slope formula. To avoid confusion, if you are asked to find the equation of a tangent line, follow the methods that we did previously.
      1. Find the slope by finding the derivative and plugging in the point
      2. Find a point on the curve (if not already given) by plugging x into the function
      3. Use point-slope to find the equation of the line
    - Note: Local approximation or tangent line approximation may also be referred to as (local) linearization. We have too many terms for the same concept. Be careful.
    - See the previous sections for examples for finding tangent lines. Some questions may read: "Find the linear approximation to the function $f(x) = \tan(x)$ at $a = \frac{\pi}{4}$." or "Find the linearization of $f(x) = \sin(2x)$ at $x = 0$."
    - Suppose $g(1) = 2$ and $g'(1) = 3$. Find the equation of the tangent line at $(1, 2)$.

Section 3.10: Theorems About Differentiable Functions

- Know the Mean Value Theorem
  - ⋆ If $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b), then there exists a number c between a and b such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.
- Know the Increasing Function Theorem (discussed in Section 2.3)
  - ⋆ If $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b), then
    (i) If $f'(x) > 0$ on (a, b), then $f(x)$ is increasing on $[a, b]$ and
    (ii) If $f'(x) \geq 0$ on (a, b), then (x) is nondecreasing on $[a, b]$.
- Know the Constant Function Theorem
  - ⋆ If $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b) and if $f'(x) = 0$ on (a, b), then $f(x)$ is constant on $[a, b]$.
- Know the Racetrack Principle
  - ⋆ If $g(x)$ and $h(x)$ are continuous on $[a, b]$ and differentiable on (a, b), and $g'(x) \leq h'(x)$ on (a, b), then
    (i) If $g(a) = h(a)$, then $g(x) \leq h(x)$ on $[a, b]$
    (ii) If $g(b) = h(b)$, then $g(x) \geq h(x)$ on $[a, b]$

Chapter 4: Using the Derivative

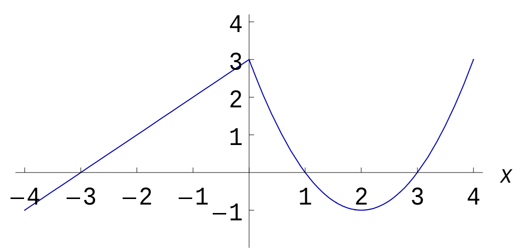
Section 4.1: Using First and Second Derivatives

- Know that $f(x)$ has a critical point wherever the first derivative is equal to 0.
  - Note: This is not the same as $f(x)$ having a max or a min at this point. It is possible that $f'(p) = 0$ and p is not a local max or min!
- Know the first-derivative test for local max/min
  - If p is a critical point and the derivative changes from neg to pos, p is a local min
  - If p is a critical point and the derivative changes from pos to neg, p is a local max
  - If p is a critical point and the derivative does not change, p is neither
- To make this a bit simpler, draw a table

$f'(x)$	+ / -	+ / -	+ / -	+ / -
$f(x)$	$\nearrow$ / $\searrow$	$\nearrow$ / $\searrow$	$\nearrow$ / $\searrow$	$\nearrow$ / $\searrow$

  - The vertical lines represent where the function has critical points
  - In each area, circle whether the derivative is positive or negative (and the shape below).
  - If $f'(x) > 0$, $f(x)$ is increasing and if $f'(x) < 0$, $f(x)$ is decreasing.
  - The arrows will give you a rough shape of the graph. From that, you can see if the critical points are local maxs, mins, or neither.
- Alternatively, you can use the second-derivative test to classify the critical points.
  - If p is a critical point and $f''(p) > 0$, then p is a local min
  - If p is a critical point and $f''(p) < 0$, then p is a local max
  - If p is a critical point and $f''(p) = 0$, then no information is known from this test
- Know that $f(x)$ may have point of inflection wherever the second derivative is equal to 0.
- Know whether or not a function is concave up or concave down (the sign of $f''(x)$)
  - Make another table like above, but for $f''(x)$ and $f(x)$
- Given a graph of $f'(x)$, know how to determine where $f(x)$ is increasing/decreasing and concave up/concave down.

- Given the graph of $f'(x)$ (below), find the interval(s) where $f(x)$ is increasing, decreasing, concave up, concave down, as well as list the x-coordinate(s) of all local max/min of $f(x)$ and list the x-coordinate(s) of the inflection points of $f(x)$.



- Know what it means for a function to have a global max/min on an interval
- Given an equation of $f(x)$, know how to determine where $f(x)$ is increasing/decreasing, concave up/concave down, as well as local/global max/mins.
  - Let $f(x) = x^3 - 9x^2 + 24x$. On what intervals is $f(x)$ increasing? Decreasing? Concave up? Concave down? Find all local max/min. Find all global max/min on the interval $[0, 7]$.

Section 4.2: Families of Curves

- Not much to say here...the problems that we will ask will be more related to Section 4.1 than anything else.

Section 4.3: Optimization

- Know that $f(x)$ has a global minimum at a point p if $f(p) \leq f(x)$ for all x.
- Know that $f(x)$ has a global maximum at a point p if $f(p) \geq f(x)$ for all x.
- Know how to find the global max/min of a function on a closed interval:
  1. First, find the critical points on the interval and their corresponding y-values.
  2. Find the y-values of the end points of the interval
  3. The largest y-value corresponds with the global max on the interval and the smallest y-value corresponds with the global min on the interval.

  - Find the absolute maximum and minimum values of $f(x) = x^3 - 9x^2 + 15x + 3$ on the interval $[0, 2]$.
  - Find the absolute maximum and minimum values of $f(x) = 3x^4 + 4x^3 - 12x^2 + 7$ on the interval $[-1, 2]$.

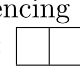
- Know that the extreme value theorem says that if you have a continuous function on a closed interval, then it will have a global maximum and a global minimum.

Section 4.4: Applications to Marginality

- Know what the cost function, revenue function and profit function are how they are related (profit(x) = revenue(x) − cost(x))
- Know what is meant by the marginal cost (the cost of making one more item)
- Know what is meant by the marginal revenue (the revenue gained from making one more item)
- Know how to maximum profit for a given scenario
  - Since profit = revenue − cost, this is just asking us to find where the profit function has a zero derivative. See Sections 4.1 and 4.3 for more on how to solve these problems.

Section 4.5: Optimization and Modeling

- This section is closely related to Section 4.3. See above for more information.
- Know the general strategy for solving optimization problems
  1. Draw a picture (if not given) of the problem and label variables.
  2. Write an equation for the expression that is to be optimized.
  3. Write an equation that uses the constraint given.
  4. Substitute the constraint into (2) to get an equation with only one variable.
  5. Take the derivative of (4).
  6. Set the derivative equal to 0 and solve for the variable.
  7. Plug the solution from (6) into the constraint equation (3) to get the other variable.
  8. Check to see that your answer makes sense, especially if you are dealing with a word problem.

- The difference of two numbers is 18. What is the smallest possible value for the product of these two numbers?
- Farmer (Old) McDonald has 1200 feet of fencing and wishes to use it to fence a rectangular plot divided into two subplots like so: . What are the dimensions of the plot with maximum area?

Section 4.6: Rates and Related Rates

- There is no algorithm for solving these types of problems, which can make them difficult to start. Here is a general strategy:

  1. Determine the variables involved in our scenario.
  2. Find some formula that relates the variables.
  3. Implicitly differentiate all variables with respect to time.
  4. Plug in the quantities you know and solve for the unknown.

  - A spherical cell is growing at a constant rate of 400 μm^3 ($1\mu m = 10^{-6} m$). At what rate is its radius increasing when the radius is 10 μm?
  - A 6 foot ladder stands leans against a wall. The foot of the ladder moves outward at a speed of 0.5ft/sec when the foot is 2 feet from the wall. At that moment, how fast is the top of the ladder falling?

Section 4.7: L'Hopital's Rule, Growth, and Dominance

- Know what L'Hopital's Rule says and when we can apply it

  ⋆ If we are considering a limit $\lim_{x \to a} \frac{f(x)}{g(x)}$ and we get $\frac{0}{0}$ or $\frac{\infty}{\infty}$, we can instead look at $\lim_{x \to a} \frac{f'(x)}{g'(x)}$ and these two limits will be equal.

  - Calculate $\lim_{x \to 0} \frac{\cos(x) - 1}{x}$

- Note: Sometimes you may need to apply L'Hopital's Rule twice

  - Determine $\lim_{x \to \infty} x^2 e^{-x}$

- Know what it means for a function $g(x)$ to dominate another function $f(x)$ as $x \to \infty$

- ⋆ We say that $g(x)$ dominates $f(x)$ as $x \to \infty$ if $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$.
  - Show that e^x dominates x^2 as $x \to \infty$.

Appendix C: Newton's Method

- Know the formula for Newton's Method: $x_i = x_{i-1} - \frac{f(x_{i-1})}{f'(x_{i-1})}$
- Given an initial guess, know how to refine your guess following the above formula.
  - There is a root near 2 of the function $f(x) = x3 - 2x - 5$. Use Newton's method twice to find a better approximation for the root.

List of Common Mistakes

- Be careful when performing addition with parentheses etc.
  - $2(x^2 + 4) = 2x^2 + 8$ but $2(x^2 + 4) \neq 2x^2 + 8$
  - $(x + y)^2 \neq x^2 + y^2$
  - $\sqrt{x+y} \neq \sqrt{x} + \sqrt{y}$
  - $\cos(x + y) \neq \cos(x) + \cos(y)$
  - In general, $f(x + y) \neq f(x) + f(y)$. (The exception is if $f(x)$ is a *linear* function.)
- Be careful not to make a cancellation error
  - $\frac{3x^2 - x}{x} = \frac{x(3x-1)}{x} = \frac{3x-1}{1} = 3x - 1$
  - However, $\frac{3x^2 - 1}{x} \neq \frac{3x-1}{1} = 3x - 1$.
- If you take a square root, don't forget the $\pm$
  - You may not need the negative term and if that is the case, say why you don't need it.
- Be careful with the property of logs

- $\ln(a+b) \neq \ln a + \ln b$; rather, only $\ln(ab) = \ln a + \ln b$
- $\dfrac{\ln a}{\ln b} \neq \ln(\dfrac{a}{b})$
- If you want to simplify something and you don't know if it is valid, plug it into your calculator (with numbers) and see if they are equal

• When you get an answer, check it to see if it makes sense

- For an exponential growth problem, if you solve for k and find that it is negative, check your work. Recall, $k > 0$ for exponential growth. (It is negative for exponential decay.)

• Don't simplify your answer if you aren't sure that you can

- And if you do, test it to make sure that it does work. *If you have any doubt, leave your answer as it is!*

So, You're Stuck on the Test. What Do You Do Now?

First of all, don't panic. If you panic, you will spiral downwards and do poorly. (It has happened to me. It is not fun.)

You could start screaming at the top of your lungs and wave your arms frantically, but don't. Rather, relax and take a deep breath. Close your eyes and think about something happy, preferably not about math. Think of the movie Happy Gilmore (if you have seen it) and *Go to your happy place.*

Now, identify what the question is asking for. Write down any formulas that you think apply from your reference sheets. Also, consider writing down a list of ideas that you will want to try.

If this hasn't led you to the answer, put the question down and move on. See if there are other problems that you do know how to solve before coming back to it.

Don't spend all your time on one problem. Leave it and come back to it!

Do another problem, maybe you will think about another way to do the previous problem.

If you are confused about what a problem is asking, you can raise your hand and we will try to help you. *We cannot tell you what to do or if you are doing it correctly, so don't ask!*

What Can I Do To Prepare?

Get a good night's sleep and eat a good meal before taking the test. Studies have shown that students do 15-25% better on exams if they get a good night's sleep (at least six uninterrupted hours of sleep) and have a good meal before their exam.

This means don't pull an all-nighter cramming for this test. Sleep instead. Your mind and body will thank you for it.

Don't study on the day of the exam, instead just look (briefly) over the material. Your studying should end the day before. You want to have a night when you can sleep and process everything that you took in.

If you study the day of the test, you won't remember as much and what you knew will start to become confused. On the test, you will make silly mistakes and kick yourself later for them. *I'm not kidding, this really does happen. Don't let it be you.*