University of California, San Diego
Department of Mathematics

Instructions

1. **Write your *Name*, *PID*, and *Section* on the front of your Blue Book.**
2. **No calculators or other electronic devices are allowed during this exam.**
3. **You may use one page of notes, but no books or other assistance during this exam.**
4. **Read each question carefully, and answer each question completely.**
5. **Write your solutions clearly in your Blue Book**
   (a) **Carefully indicate the number and letter of each question.**
   (b) **Present your answers in the same order they appear in the exam.**
   (c) **Start each question on a new page.**
6. **Show all of your work; no credit will be given for unsupported answers.**

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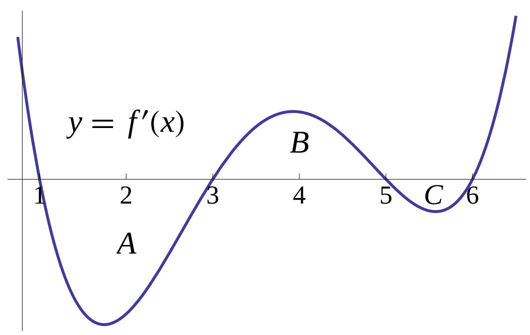
1. (10 points) The graph below is the graph of $y = f'(x)$, the ***derivative*** of the function f.



The area of region A is 6, the area of region B is 4, and the area of region C is 1.

(a) If $f(1) = 3$, what is $f(6)$?

(b) Find the absolute maximum value of $f(x)$ on the interval $[1, 6]$ and indicate at which x value it occurs.

2. Compute the following indefinite integrals:

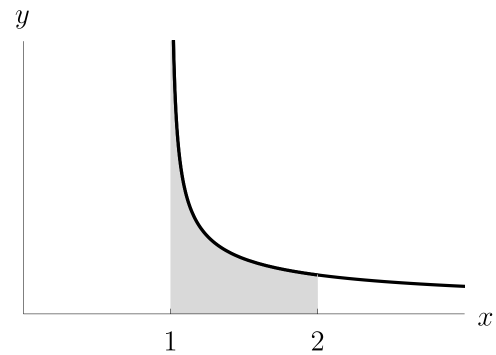
(a) (10 points) $\displaystyle\int x^2 e^x \, dx$

(b) (10 points) $\displaystyle\int \frac{1}{(4+z^2)^{3/2}} \, dz$

(c) (10 points) $\displaystyle\int \frac{3x+1}{x(x^2-1)} \, dz$

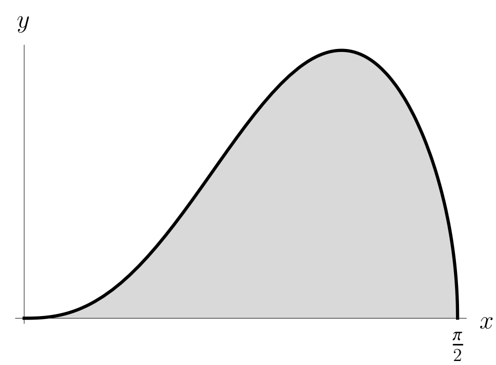
(Please turn over.)

3. (10 points) Find the area of the region bounded by $y = \dfrac{1}{\sqrt{x-1}}$, $x = 1$, $x = 2$, and the x-axis.



4. (10 points) For the improper integral, state whether or not it converges. Justify your answer: $\displaystyle\int_1^\infty \frac{2z}{4+z^3}\,dz$

5. (10 points) The figure below shows the curve of $y = \sqrt{\sin^5 x \cos x}$ over the interval $[0, \pi/2]$ and the region under the curve. Calculate the volume of the solid *obtained by rotating the shaded region about the* ***x*-axis**.



6. (10 points) An object is heated to a temperature of 100° F and then placed in a room held at a constant temperature of 60° F. Let $H(t)$ be the temperature of the object t minutes after being placed in the room. According to Newton's Law of Cooling, the rate of change in H over time is given by the differential equation:

$$\frac{dH}{dt} = k(H - 60),$$

where k is a constant.

(a) Solve the differential equation subject to the initial conditions given in the statement of the problem. (Your answer will have a k in it.)

(b) Ten minutes after the object was placed in the room, it had a temperature of 80° F. Find the *exact* value of the constant k. (Your answer will include a logarithm. This is as it should be.)

(c) Find $\lim_{t\to\infty} H(t)$. (Justify your answer.)

(This exam is worth 80 points.)