Name: \_\_\_\_\_ PID: \_\_\_\_\_

- Print your NAME on every page and write your PID in the space provided above.
- Show all of your work in the spaces provided. No credit will be given for unsupported answers, even if correct.
- No calculators, tablets, phones, or other electronic devices are allowed during this exam. You may use one page of handwritten notes, but no books or other assistance.

## Do not turn the page until instructed to do so.

## Question Zero.

Follow the instructions on this exam and any additional instructions given during the exam.

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin(\theta)$	0	1/2	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\cos(\theta)$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	1/2	0
$\tan(\theta)$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	_



(6 pt) 1. Determine whether the integral converges or diverges and evaluate it if it converges:

$$\int_{-1}^1 \frac{1}{u^2} \, du$$



(6 pt) 2. The base of a solid object is the region in the first quadrant  $(x \ge 0, y \ge 0)$  bounded by  $y = \sqrt{9 - x^2}$  and the *x*-axis and *y*-axis. Cross-sections perpendicular to the *x*-axis are squares. Find the volume of the solid object.



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(6 pt) 4. Find g'(x) if  $g(x) = \int_{1}^{2x} \frac{e^{7t}}{t^2} dt$ 



(6 pt) 5. Evaluate the indefinite integral:  $\int \frac{-x+7}{x^2+x-6} dx$ 



(8 pt) 6. Use a trigonometric substitution to evaluate the definite integral:  $\int_0^1 \frac{1}{\sqrt{4-x^2}} dx$ 

(6 pt) 7. Find the solution of the differential equation that satisfies

$$\frac{dy}{dx} = \frac{\ln(x)}{xy}, \quad y(1) = 2$$



- (6 pt) 8. A bacteria culture initially contains 500 cells and grows at a rate proportional to its size After 2 hours, the population increases to 2000 cells.
  - (a) Find a formula for the number of bacteria after t hours.
  - (b) When will the population reach 1 million  $(=10^6)$  cells? Your answer may be written in terms of a logarithm.

(5 pt) 9. Match the given differential equations to their direction fields. For each differential equation given below, write the UPPER CASE letter corresponding to the direction field in the blank space provided.

