Name: $\qquad$ PID: $\qquad$

- Print your NAME on every page and write your PID in the space provided above.
- Show all of your work in the spaces provided. No credit will be given for unsupported answers, even if correct.
- No calculators, tablets, phones, or other electronic devices are allowed during this exam. You may use one page of handwritten notes, but no books or other assistance.


## Do not turn the page until instructed to do so.

## Question Zero.

Follow the instructions on this exam and any additional instructions given during the exam.

| $\theta$ | 0 | $\pi / 6$ | $\pi / 4$ | $\pi / 3$ | $\pi / 2$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\sin (\theta)$ | 0 | $1 / 2$ | $\sqrt{2} / 2$ | $\sqrt{3} / 2$ | 1 |
| $\cos (\theta)$ | 1 | $\sqrt{3} / 2$ | $\sqrt{2} / 2$ | $1 / 2$ | 0 |
| $\tan (\theta)$ | 0 | $1 / \sqrt{3}$ | 1 | $\sqrt{3}$ | - |

(6 pt) 1. Determine whether the integral converges or diverges and evaluate it if it converges:

$$
\int_{-1}^{1} \frac{1}{u^{2}} d u
$$

(6 pt) 2. The base of a solid object is the region in the first quadrant ( $x \geq 0, y \geq 0$ ) bounded by $y=\sqrt{9-x^{2}}$ and the $x$-axis and $y$-axis. Cross-sections perpendicular to the $x$-axis are squares. Find the volume of the solid object.
(6 pt) 3. Find the area of the region bounded by $y=x e^{-2 x}, x=0, x=3$, and $y=0$.

(6 pt) 4. Find $g^{\prime}(x)$ if $g(x)=\int_{1}^{2 x} \frac{e^{7 t}}{t^{2}} d t$
$(6 \mathrm{pt})$ 5. Evaluate the indefinite integral: $\int \frac{-x+7}{x^{2}+x-6} d x$
(8 pt) 6. Use a trigonometric substitution to evaluate the definite integral: $\int_{0}^{1} \frac{1}{\sqrt{4-x^{2}}} d x$
(6 pt) 7. Find the solution of the differential equation that satisfies

$$
\frac{d y}{d x}=\frac{\ln (x)}{x y}, \quad y(1)=2
$$

( 6 pt ) 8. A bacteria culture initially contains 500 cells and grows at a rate proportional to its size After 2 hours, the population increases to 2000 cells.
(a) Find a formula for the number of bacteria after $t$ hours.
(b) When will the population reach 1 million $\left(=10^{6}\right)$ cells? Your answer may be written in terms of a logarithm.
(5 pt) 9. Match the given differential equations to their direction fields. For each differential equation given below, write the UPPER CASE letter corresponding to the direction field in the blank space provided.
(i) $\quad \frac{d y}{d x}=x y$
(ii) $-\frac{d y}{d x}=-y$
(iii) $\frac{d y}{d x}=\sin (x)$
(iv) $-\frac{d y}{d x}=y+x$
(v) $-\frac{d y}{d x}=\sin (y)$
(A)

(B)

(C)

(D)

(E)

(F)


