

$$1. \int \frac{\sin \sqrt{x}}{\sqrt{x}} dx = 2 \int (\sin \sqrt{x}) \cdot \left[ \frac{1}{2} \frac{1}{\sqrt{x}} dx \right] \quad u = \sqrt{x} = x^{1/2} \quad du = \frac{1}{2} x^{-1/2} dx$$

$$= 2 \int \sin u du = -2 \cos u + C = \boxed{-2 \cos \sqrt{x} + C}$$

$$2. \int \frac{x+3}{(x-1)(x^2+1)} dx = \int \left( \frac{A}{x-1} + \frac{Bx+C}{x^2+1} \right) dx$$

$$\left. \begin{aligned} x+3 &= A(x^2+1) + (Bx+C)(x-1) \\ &= Ax^2 + A + Bx^2 + Cx - Bx - C \\ &= (A+B)x^2 + (C-B)x + (A-C) \end{aligned} \right\} \begin{aligned} A+B &= 0 & C-B &= 1 & A-C &= 3 \\ B &= -A & C+A &= 1 & A+3+C & \\ & & C+(3+1) &= 1 & & \\ & & 2C &= -2 & & \\ & & C &= -1 & A &= 2 \end{aligned}$$

$$= \int \left( \frac{2}{x-1} + \frac{-2x-1}{x^2+1} \right) dx = \int \left( \frac{2}{x-1} - \frac{2x}{x^2+1} - \frac{1}{x^2+1} \right) dx \quad B=-2 \quad C=-1 \quad A=2$$

$$= \boxed{2 \ln|x-1| - \ln(x^2+1) - \arctan(x) + C}$$

$$3. \int_1^{\infty} x e^{-x} dx = \lim_{t \rightarrow \infty} \int_1^t x e^{-x} dx = \lim_{t \rightarrow \infty} \left[ -x e^{-x} + \int e^{-x} dx \right]_1^t \quad \begin{aligned} u &= x \quad du = e^{-x} dx \\ dv &= e^{-x} \quad v = -e^{-x} \end{aligned}$$

$$= \lim_{t \rightarrow \infty} \left[ -x e^{-x} - e^{-x} \right]_1^t = \lim_{t \rightarrow \infty} \left[ \underbrace{(-t e^{-t} - e^{-t})}_{\rightarrow 0} - \underbrace{(-e^{-1} - e^{-1})}_{-2/e} \right]$$

$$= \boxed{\frac{2}{e}}$$

Note:  $\lim_{t \rightarrow \infty} t e^{-t} = \lim_{t \rightarrow \infty} \frac{t}{e^t} \stackrel{\text{H}}{\rightarrow 0} = \lim_{t \rightarrow \infty} \frac{1}{e^t} = 0$

$$4. \int \frac{1}{x^2+2x+2} dx = \int \frac{1}{(x^2+2x+1)+1} dx = \int \frac{1}{(x+1)^2+1} dx$$

$$= \int \frac{1}{u^2+1} du = \arctan(u) + C \quad \boxed{u=x+1 \quad du=dx}$$

$$= \boxed{\arctan(x+1) + C}$$

OR: Trig substitution:  $x+1 = \tan \theta \quad dx = \sec^2 \theta d\theta$

$$\text{Answer} = \int \frac{1}{\tan^2 \theta + 1} \cdot \sec^2 \theta d\theta = \int \frac{1}{\sec^2 \theta} \cdot \sec^2 \theta d\theta = \int 1 d\theta$$

$$= \theta + C = \boxed{\arctan(x+1) + C}$$