1. $\int \frac{(\ln z)^2}{z}\,dz$ $u = \ln z$, $du = \frac{1}{z}\,dz$

$= \int \underbrace{(\ln z)^2}_{u^2} \cdot \underbrace{\frac{1}{z}\,dz}_{du}$

$= \int u^2\,du = \frac{u^3}{3} + C$

$= \frac{(\ln z)^3}{3} + C$

2. $\int \frac{e^x}{2+e^x}\,dx$ $u = 2+e^x$, $du = e^x dx$

$= \int \underbrace{\frac{1}{2+e^x}}_{1/u} \cdot \underbrace{e^x\,dx}_{du}$

$= \int \frac{1}{u}\,du = \ln|u| + C$

$= \ln(2+e^x) + C$

3. $\int y\,(y+3)^{1/2}\,dy$ $u = y$, $dv = (y+3)^{1/2}$, $du = dy$, $v = \frac{2}{3}(y+3)^{3/2}$

$= \frac{2}{3}y(y+3)^{3/2} - \int \frac{2}{3}(y+3)^{3/2}\,dy$

$= \frac{2}{3}y(y+3)^{3/2} - \frac{4}{15}(y+3)^{5/2} + C$

4. $\int \frac{x+1}{x^3+x}\,dx = \int \frac{x+1}{x(x^2+1)}\,dx$

$\frac{x+1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1} = \frac{A(x^2+1)+(Bx+C)x}{x(x^2+1)}$

$x+1 = Ax^2 + A + Bx^2 + Cx$

$= (A+B)x^2 + Cx + A$

$A+B = 0$ $C = 1$ $A = 1$

$B = -A = -1$

Integral $= \int \left(\frac{1}{x} + \frac{-x+1}{x^2+1} \right) dx$

$= \int \left(\frac{1}{x} - \frac{x}{x^2+1} + \frac{1}{x^2+1} \right) dx$

$= \ln|x| \underbrace{- \frac{1}{2}\ln(x^2+1)}_{\text{Use substitution to get this one}} + \arctan(x) + C$

5. $\int \frac{dz}{(4-z^2)^{3/2}}$ $z = 2\sin\theta$ $dz = 2\cos\theta\,d\theta$

$= \int \frac{2\cos\theta}{(4-4\sin^2\theta)^{3/2}}\,d\theta = \int \frac{2\cos\theta}{[4(1-\sin^2\theta)]^{3/2}}\,d\theta$

$= \int \frac{2\cos\theta}{[4\cos^2\theta]^{3/2}}\,d\theta = \int \frac{2\cos\theta}{(2\cos\theta)^3}\,d\theta$

$= \int \frac{2\cos\theta}{8\cos^3\theta}\,d\theta = \int \frac{1}{4}\frac{1}{\cos^2\theta}\,d\theta$

$= \frac{1}{4}\int \sec^2\theta\,d\theta = \frac{1}{4}\tan\theta + C$

$= \frac{1}{4}\tan\left(\arcsin\left(\frac{z}{2}\right)\right) + C = \frac{1}{4}\frac{z}{\sqrt{4-z^2}} + C$

6. $\int \frac{1}{x^2+4x+13}\,dx = \int \frac{1}{(x^2+4x+4)+9}\,dx$

$= \int \frac{1}{(x+2)^2+9}\,dx = \frac{1}{9}\int \frac{1}{\left(\frac{x+2}{3}\right)^2+1}\,dx$

$= \frac{1}{9}\int \frac{1}{u^2+1}\cdot 3\,du$ $\boxed{u = \frac{x+2}{3} \quad du = \frac{dx}{3}}$

$= \frac{1}{3}\arctan u + C = \frac{1}{3}\arctan\left(\frac{x+2}{3}\right) + C$

<u>OR</u> Integral $= \int \frac{1}{(x+2)^2+9}\,dx$

$x+2 = 3\tan\theta$ $dx = 3\sec^2\theta\,d\theta$

$= \int \frac{1}{(3\tan\theta)^2+9}\cdot 3\sec^2\theta\,d\theta$

$= \int \frac{3\sec^2\theta}{9\tan^2\theta+9}\,d\theta = \int \frac{3\sec^2\theta}{9(\tan^2\theta+1)}\,d\theta$

$= \int \frac{3\sec^2\theta}{9\sec^2\theta}\,d\theta = \int \frac{1}{3}\,d\theta$

$= \frac{1}{3}\theta + C$

$= \frac{1}{3}\arctan\left(\frac{x+2}{3}\right) + C$

<u>Note</u>: $x+2 = 3\tan\theta$

$\Rightarrow \frac{x+2}{3} = \tan\theta$

$\Rightarrow \arctan\left(\frac{x+2}{3}\right) = \theta$

7. $\int_2^\infty \frac{dx}{x\ln x}\,dx$ $u = \ln x$ $du = \frac{1}{x}dx$; $x = \infty$, $u = \infty$; $x = 2$, $u = \ln 2$

$= \int_{\ln 2}^\infty \frac{1}{u}\,du$

$= \lim_{b\to\infty} \int_{\ln 2}^b \frac{1}{u}\,du = \lim_{b\to\infty} \ln|u| \Big|_{\ln 2}^b$

$= \lim_{b\to\infty} \ln(b) - \ln(\ln 2) = \infty$

(Diverges)

8. $\int_1^2 \frac{dx}{x \ln x}$ $\quad u = \ln x \quad du = \frac{1}{x}dx$

$x=1 \quad x=2$

$u=0 \quad u=\ln 2$

$= \int_0^{\ln 2} \frac{1}{u}\,du$

$= \lim_{a\to 0^+} \int_a^{\ln 2} \frac{1}{u}\,du = \lim_{a\to 0^+} \ln|u| \Big|_a^{\ln 2}$

$= \lim_{a \to 0^+} \ln(\ln 2) - \underbrace{\ln a}_{\to -\infty} = \infty$

(Diverges)

9. $\int_1^\infty \frac{1}{1+x}\,dx \qquad 1+x < x+x = 2x$

$\frac{1}{1+x} > \frac{1}{2x}$

$\int_1^\infty \frac{1}{1+x}\,dx > \int_1^\infty \frac{1}{2x}\,dx = \infty$

Diverges

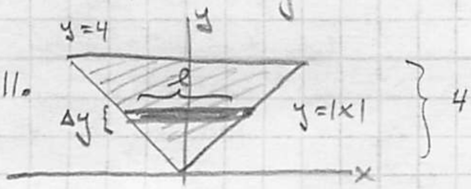
10. $\int_0^\infty \frac{dz}{e^z + 2^z} \qquad e^z + 2^z > e^z$

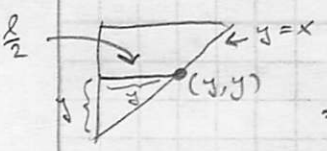
$\Rightarrow \frac{1}{e^z+2^z} < e^{-z}$

$\int_0^\infty \frac{dz}{e^z+2^z} < \int_0^\infty e^{-z}\,dz < \infty$

Converges

11.





$\frac{l}{2} = y$

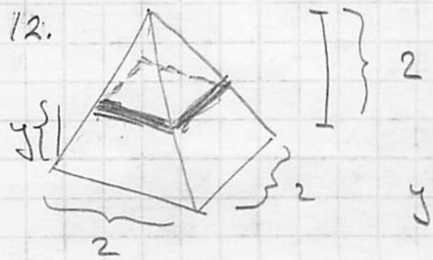
$\Rightarrow l = 2y$

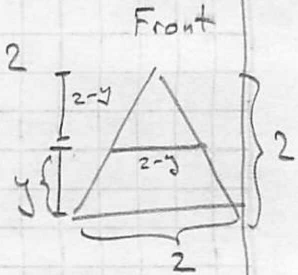
Area of slice $\approx 2y\,\Delta y$

Area $= \int_0^4 2y\,dy = y^2\Big|_0^4$

$= 16$

12.





Volume of slice $\approx (2-y)^2\,\Delta y$

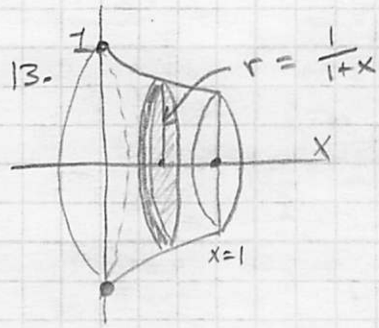
Volume $= \int_0^2 (2-y)^2\,dy$

$= \int_0^2 (4 - 4y + y^2)\,dy$

$= 4y - 2y^2 + \frac{y^3}{3}\Big|_0^2$

$= 8 - 8 + 8/3 = \boxed{8/3}$

13.



Volume of slice $\approx \pi\left(\frac{1}{1+x}\right)^2 \Delta x$

Volume $= \int_0^1 \pi\left(\frac{1}{1+x}\right)^2 dx$

$= \pi \int_0^1 \frac{1}{(1+x)^2}\,dx \qquad u = 1+x \quad du = dx$

$x=0 \quad x=1$

$u=1 \quad u=2$

$= \pi \int_1^2 \frac{1}{u^2}\,du$

$= \pi\left(-\frac{1}{u}\right)\Big|_1^2 = \pi\left(-\frac{1}{2}+1\right)$

$= \frac{\pi}{2}$