Fall 2015 Total Points: 60 Math 10B Exam 3C

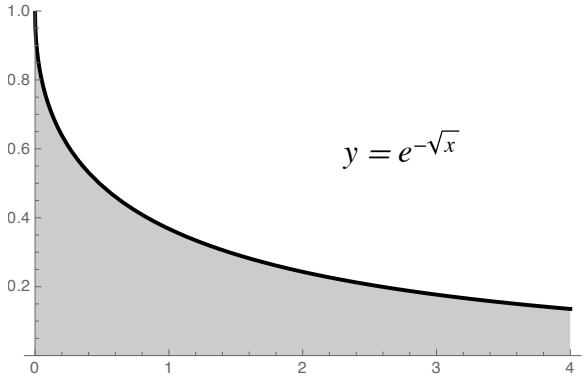
Name: _____ PID: _____

- Print your *NAME* on every page and write your *PID* in the space provided above.
- Show all of your work in the spaces provided. No credit will be given for unsupported answers, even if correct.
- No calculators, tablets, phones, or other electronic devices are allowed during this exam. You may use one page of handwritten notes, but no books or other assistance.

(1 pt) 0. Follow the instructions on this exam and any additional instructions given during the exam.

(5 pt) 1. If F is an antiderivative of f and $F(x) = e^{\sin(x)}$, then compute $\displaystyle\int_0^x f(s)\,ds$.

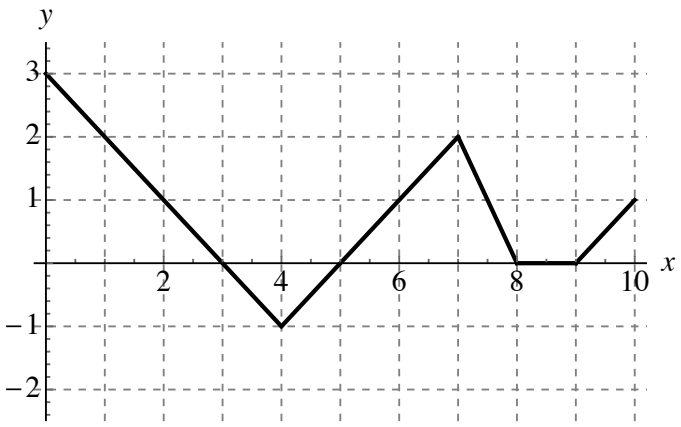
(6 pt) 2. The shaded region below is the region in the first quadrant bounded by $y = e^{-\sqrt{x}}$ and $x = 4$.



(a) Set up, but do not evaluate, an integral that will give the average value of $f(x) = e^{-\sqrt{x}}$ over the interval $[0, 4]$.

(b) Write down a function F that is an antiderivative of f with the property that $F(1) = 5$. Your answer may be written in terms of an integral.

(6 pt) 3. Below is the graph of the function f:

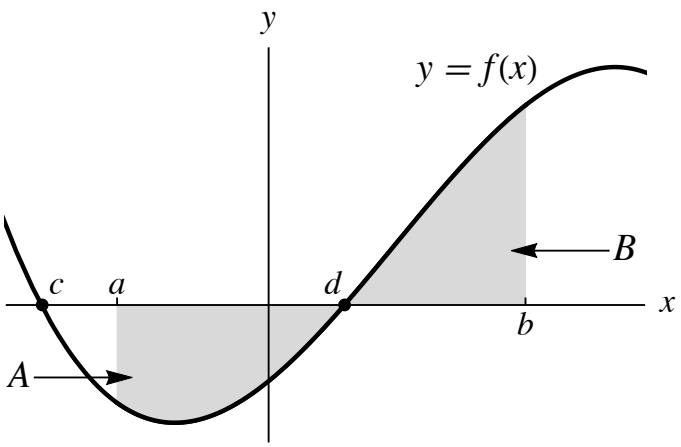


(a) Compute $\int_0^9 f(x)\,dx$.

(b) What is the average value of f over the interval $[0, 9]$?

(c) Compute $\int_0^9 \frac{f(\sqrt{x})}{\sqrt{x}}\,dx$.

(6 pt) 4. The following is a graph of the function f. Assume the area of region A is 5 and the area of region B is 6.



Let $F(x) = \int_a^x f(t)\,dt$.

(a) Give the values of $F(a)$, $F(c)$, and $F(b)$.

(b) Is $F(d)$ a positive number, a negative number, or zero?

(c) Compute $\int_b^a |f(t)|\,dt$.

(d) Is F concave up, concave down, or does it have a point of inflection at $x = c$?

(6 pt) 5. Evaluate the integral: $\int_0^2 \sqrt{4-x^2}\,dx$

(6 pt) 6. Evaluate the integral: $\int 3x^2 \ln(x)\, dx$

(6 pt) 7. Evaluate the improper integral or show that it diverges: $\int_0^\infty xe^{x^2}\,dx$

(6 pt) 8. Find the general solution to the differential equation:

$$\frac{dy}{dt} = -4\sin(2t) + 6t$$

(6 pt) 9. Find an explicit solution $y = f(x)$ to the initial value problem:

$$\frac{dy}{dx} = \frac{6xy}{\ln(y)}, \qquad y(0) = e^2$$

(6 pt) 10. Find the second degree Taylor polynomial for the function $\ln(x)$ centered at $a = 3$.