Game: Draw one card from a standard poker deck.

Rules: If you draw an ace, you win $5
If you draw a face card (Jack, Queen, King), you lose $5
If you draw any other card (2-10), you neither win nor lose.

Question: If you play this game 100 times, what is the probability that your total winnings are nonnegative?

Solution: Let \( W_i \) = winnings on \( i \)th play.
\[
W_i = \begin{cases} 
+5 & \text{if } A \\
0 & \text{if } 1-10 \\
-5 & \text{if } J, Q, K
\end{cases}
\]

\[
P(W_i = 5) = \frac{4}{13} \\
P(W_i = 0) = \frac{1}{2} \\
P(W_i = -5) = \frac{3}{13}
\]

\[
\mu = \mathbb{E}(W_i) = (5)(\frac{4}{13}) + (0)(\frac{1}{13}) + (-5)(\frac{3}{13}) = -\frac{10}{13} \approx -0.77
\]

\[
\mathbb{E}(W_i^2) = (5)^2(\frac{4}{13}) + (0)^2(\frac{1}{13}) + (-5)^2(\frac{3}{13}) = \frac{100}{13}
\]

\[
\sigma^2 = \text{Var}(W_i) = \left(\frac{100}{13}\right) - \left(-\frac{10}{13}\right)^2 = \frac{1200}{169} \approx 7.1 = (2.66)^2
\]

Let \( S_{100} = W_1 + \cdots + W_{100} = \text{total winnings} \quad (n = 100) \)

By the Central Limit Theorem,
\[
S_{100} \approx N(\mu, n\sigma^2) = N(-77, 710) = N(-77, (26.6)^2)
\]

\[
P(S_{100} \geq 0) \approx P(Z \geq \frac{-77}{26.6}) = P(Z \geq -2.89)
\]

\[
= P(Z \geq 2.89) = 1 - P(Z < 2.89) = 1 - 0.9981 = 0.0019
\]

Conclusion: If you play this game 100 times, your chances of not losing money (overall) are 0.19%.

Real conclusion: Don't play this game.

Comment: In class, I used \( \sigma = 7.1 \). (I forgot the square root.)