Notes:

- These are examples of the types of questions that will be on the *second half* of the exam.
- The actual exam will consist of 5 questions from the first half of the quarter and five questions from the second half of the quarter.
- You do not need to evaluate factorials such as 4! or expressions like $\binom{5}{3}$.
- The more work you show and explanation you give, the more partial credit we will be able to assign.
- For the exam, you will be given a table of values for the standard normal distribution.
- You will not have a calculator on the exam, so you should not use one when doing these problems.

1. The jointly continuous random variables X and Y have joint probability density function $f_{X,Y}(x,y) = c(2x+y)$ for $0 \le x \le 1$ and $0 \le y \le 2$.
   (a) Find the value of c.
   (b) Compute the marginal PDFs $f_X(x)$ and $f_Y(y)$.
   (c) Compute the joint cumulative distribution function $F_{X,Y}(u,v)$.

2. The jointly continuous random variables X and Y have joint cumulative distribution function $F_{X,Y}(x,y) = xy$, where $0 \le x, y \le 1$. Find the joint probability density function $f_{X,Y}(x,y)$. Are X and Y independent?

3. The jointly continuous random variables X and Y have joint probability density function $f_{X,Y}(x,y) = 6e^{-2x-3y}$ for $x, y > 0$. Show that X and Y are independent. Find $\mathrm{E}(X+Y)$ and $\mathrm{Var}(X+Y)$. (*Hint:* Use the independence. What do you know about exponential random variables?)

4. The jointly continuous random variables X and Y have joint probability density function $f_{X,Y}(x,y) = 6e^{-2x-3y}$ for $x, y > 0$. Without using the formula from Section 3.8, find $f_{Y/X}(w)$. (*Hint:* Start by finding $F_{Y/X}(w)$.)

5. Suppose that Y is a nonnegative random variable with probability density function $f_Y(y)$. Let $W = \sqrt{Y}$ and find $f_W(w)$. Your answer will be in terms of the PDF for Y.

6. Suppose that IQ scores are distributed normally with mean $\mu = 100$ and standard deviation $\sigma = 16$. (Notice that we are given the standard deviation and *not* the variance.)
   (a) Compute the probability that a randomly selected person will have an IQ score greater than 104. (*Note:* You can do this without a calculator.)
   (b) Suppose 5 people are randomly selected. Write a formula that will allow us to compute the probability that at least 4 of them have an IQ score greater than 104. (Do not compute this probability.)

7. A random variable X has unknown mean μ and standard deviation $\sigma = 5$. Suppose 100 independent random measurements are made, resulting in an observed average of $\overline{X}$.
   (a) Use the attached table to solve $P(-c \le Z \le c) = 0.80$ for c. (*Note:* $F_Z(-c) = 1 - F_Z(c)$.)
   (b) Use the result of (a) to construct a *formula* for an 80% confidence interval for μ.
   (c) If 50 such intervals are constructed, how many should you expect will contain μ?
   (d) What is the margin of error for this sampling?

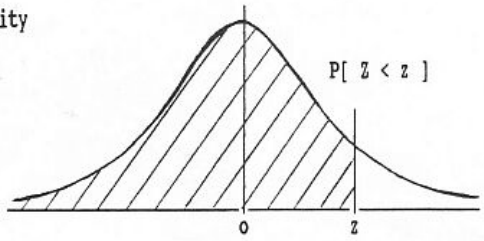
8. A continuous random variable X has probability density function $f(x;\theta) = 2^{\theta}\theta x^{-(\theta+1)}$ for $x \geq 2$ and $\theta > 1$, where θ is an unknown parameter.
   (a) Find the maximum likelihood estimator $\hat{\theta}$, assuming a sample size of n.
   (b) Show that $\frac{1}{\hat{\theta}}$ is an unbiased estimator for $\frac{1}{\theta}$. (*Note:* This involves integration by parts.)

STANDARD STATISTICAL TABLES

1. Areas under the Normal Distribution

The table gives the cumulative probability up to the standardised normal value z

i.e. $P[\,Z < z\,] = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2}Z^2)\, dZ$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5159	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7854
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8804	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9865	0.9868	0.9871	0.9874	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9980	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
z	3.00	3.10	3.20	3.30	3.40	3.50	3.60	3.70	3.80	3.90
P	0.9986	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000