Math 20A



University of California, San Diego Department of Mathematics

Instructions

- 1. Write your Name, PID, Section, and Exam Version on the front of your Blue Book.
- 2. No calculators or other electronic devices are allowed during this exam.
- 3. You may use one page of notes, but no books or other assistance during this exam.
- 4. Read each question carefully, and answer each question completely.
- 5. Write your solutions clearly in your Blue Book.
 - (a) Carefully indicate the number and letter of each question and question part.
 - (b) Present your answers in the same order they appear in the exam.
 - (c) Start each problem on a new page.
- 6. Show all of your work. No credit will be given for unsupported answers, even if correct.
- 7. Turn in your exam paper with your Blue Book.

DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO

Question Zero:

0. (2 points) Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

1. (6 points) Evaluate the following limits or state that they do not exist (DNE).

(a)
$$\lim_{\theta \to \frac{\pi}{2}} \left[\sec(\theta) - \tan(\theta) \right]$$
 (b)
$$\lim_{x \to 0} \frac{\ln(1+x^2)}{x\sin(3x)}$$

2. (6 points) Compute the derivative of each of the following functions.

(a)
$$f(x) = 10\sqrt{x+\sqrt{x}} + e^3$$
 (b) $g(x) = \frac{\arctan(x)}{\ln(x)} + \frac{x^4}{4}$

- 3. (6 points) Use the Linear Approximation to determine which is larger of $\sqrt{4.1} \sqrt{4}$ or $\sqrt{9.1} \sqrt{9}$.
- 4. (6 points) The base x of a right triangle is increasing in length at a rate of 4 cm/s, while the height remains constant at 20 cm. How fast is the angle θ changing when the base x is 20 cm in length?



- 5. (6 points) Let $f(x) = x^3 x$.
 - (a) What are the absolute maximum and absolute minimum values of f(x) on the interval [-1, 1]?
 - (b) What are the absolute maximum and absolute minimum values of f(x) on the interval [-2, 2]?
- 6. (6 points) Compute the following integrals.

(a)
$$\int_{-\pi}^{\pi} \left[3 + \cos(x) \right] dx$$
 (b) $\int \frac{1 + 4x + x^2}{x} dx$

7. (6 points) Compute $\frac{d}{dx} \int_{-x}^{x} e^{t^2} dt$.

8. (4 points) The following is a graph of the function f.



- (a) If $\int_{a}^{b} f(x) dx > 0$, then which is larger: The area of region A or the area of region B?
- (b) Let $F(x) = \int_{a}^{x} f(t) dt$. On what interval or intervals is F decreasing? Either give your answer using interval notation, or state that there is not in enough information given to solve the problem and explain why not.
- 9. (6 points) Suppose that $F(x) = e^{\sin(x)}$ and suppose that F'(x) = f(x).

(a) Compute
$$\frac{d}{dx}F(x)$$
. (b) Compute $\int_0^x f(s) \, ds$.

10. (6 points) Find the equation of the tangent line to $(x^2 + y^2)^2 = 9(x^2 - y^2)$ at the point $(\sqrt{2}, 1)$.