Math 20A



University of California, San Diego Department of Mathematics

Instructions

- 1. Write your Name, PID, Section, and Exam Version on the front of your Blue Book.
- 2. No calculators or other electronic devices are allowed during this exam.
- 3. You may use one page of notes, but no books or other assistance during this exam.
- 4. Read each question carefully, and answer each question completely.
- 5. Write your solutions clearly in your Blue Book.
 - (a) Carefully indicate the number and letter of each question and question part.
 - (b) Present your answers in the same order they appear in the exam.
 - (c) Start each problem on a new page.
- 6. Show all of your work. No credit will be given for unsupported answers, even if correct.
- 7. Turn in your exam paper with your Blue Book.
- 0. (1 point) Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.
- 1. Compute $\frac{dy}{dx}$ for each of the following.
 - (a) (4 points) $y = \tan(\sqrt{x})$.
 - (b) (4 points) $y = \ln(\arctan(x))$.
 - (c) (4 points) $y = \log_2(x^3 + 1)$.
- 2. (10 points) Use implicit differentiation to find the equation of the tangent line at the point (1,0) to the curve given by the equation

$$x^2 e^y + \sin(y^2) = 1 + 4y.$$

- 3. (8 points) Let $g(x) = 3x^4 x^6$. Find the absolute maximum and absolute minimum values of g on the closed interval $[-1, \sqrt{3}]$ and identify the x values corresponding to the extremal values.
- 4. The kinetic energy E of a rocket of mass m, traveling at velocity v is given by the formula

$$E = \frac{1}{2}mv^2.$$

- (a) (2 points) Find the rate of change of E with respect to m. (Assume the velocity is constant.)
- (b) (2 points) A rocket engine works by ejecting mass out the back of the rocket to generate forward momentum. Suppose the resulting mass of the rocket at time t is

$$m(t) = 3t^2 + t.$$

Assuming the rocket maintains a constant velocity, find the rate of change of E with respect to t.