University of California, San Diego
Department of Mathematics

Instructions

1. **Write your *Name*, *PID*, *Section*, and *Exam Version* on the front of your Blue Book.**
2. **No calculators or other electronic devices are allowed during this exam.**
3. **You may use one page of notes, but no books or other assistance during this exam.**
4. **Read each question carefully, and answer each question completely.**
5. **Write your solutions clearly in your Blue Book.**
   (a) **Carefully indicate the number and letter of each question and question part.**
   (b) **Present your answers in the same order they appear in the exam.**
   (c) **Start each problem on a new page.**
6. **Show all of your work. No credit will be given for unsupported answers, even if correct.**
7. **Turn in your exam paper with your Blue Book.**

DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO

Question Zero:

0. (1 point) Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

(This exam is worth 70 points.)

1. (9 points) Evaluate the following limits or state that they do not exist (DNE).
   (a) $\lim_{\theta \to 0} (\sec\theta - \tan\theta)$
   (b) $\lim_{x \to 0} (\cos x)^{\frac{1}{x^2}}$
   (c) $\lim_{\theta \to 0} \dfrac{\cos(8\theta)\tan(8\theta)}{\theta}$
2. (8 points) Use the linearization of $f(x) = \arctan(x)$ at $a = 1$ to approximate $\arctan(1.1)$.
3. (9 points) Compute $\dfrac{dy}{dx}$ for each of the following.
   (a) $y = x^3 3^x$
   (b) $y = \sqrt{x^2 + \sqrt{x^2+1}}$
   (c) $y = x^{\cos x}$
4. (8 points) Find the equation of the tangent line to the graph of $x^2 + xy + y^2 = 7$ at $(3, -2)$.
5. (10 points) A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a single-strand electric fence. Using 400 meters of wire, what is the largest rectangular region that can be enclosed? Give both the dimensions and the area.
6. (10 points) Let $f(x) = xe^{-x^2}$.
   (a) Identify the horizontal asymptotes of f.
   (b) Identify the intervals over which f is increasing/decreasing.
   (c) Find all critical points and label each as belonging to a local maximum, local minimum, or neither.
   (d) Identify the intervals over which f is concave down/concave up.
   (e) List the x-coordinates for all points of inflection of f.
7. (10 points) Let $f(x) = (x-1)^2$ on the interval $[-2, 2]$.
   (a) Compute R_2 and L_2 for f on $[-2, 2]$.
   (b) Compute the exact value of $\int_{-2}^{2} f(x)\,dx$ using the Fundamental Theorem of Calculus.
8. (5 points) Compute the derivative of the function $G(x) = \displaystyle\int_3^{1/x} e^{t^2}\,dt$.