Name: _____ **PID:** _____

TA: _____ **Sec. No:** _____ **Sec. Time:** _____

Math 20B.
Final Examination
June 11, 2010

Turn off and put away your cell phone.
No calculators or any other devices are allowed on this exam.
You may use one page of notes, but no books or other assistance on this exam.
Read each question carefully, answer each question completely, and show all of your work.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
If any question is not clear, ask for clarification.

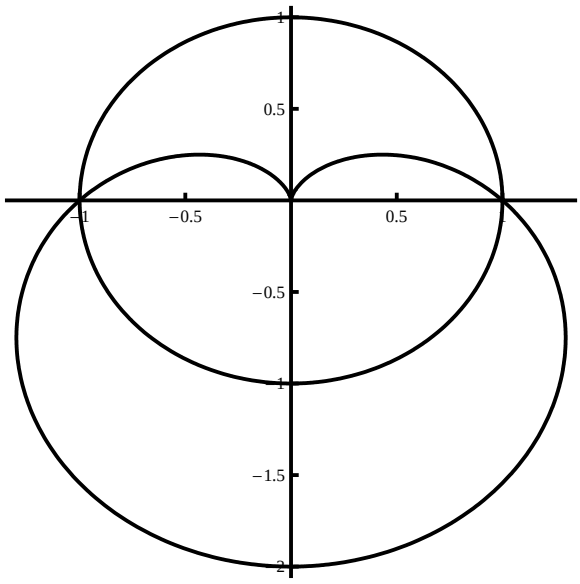
#	Points	Score
1	8	
2	6	
3	6	
4	8	
5	6	
6	6	
7	6	
8	6	
9	10	
Σ	62	

1. Let $f(x) = \dfrac{x^2 + x - 1}{(x-2)(x^2+1)}$.

(a) (4 points) Find the partial fraction decomposition of $f(x)$.

(b) (4 points) Find $\displaystyle\int f(x)\,dx$.

2. (6 points) Find the area of the region that lies *inside* the circle $r = 1$ and *outside* the curve $r = 1 - \sin(\theta)$.



3. (6 points) Using *complex exponentials*, compute $\int e^x \cos^2(x)\, dx$. You need not simplify the result and may leave it in complex exponential form.

4. (a) (4 points) Determine whether the integral $\int_0^1 \frac{1}{x+\sqrt{x}}\,dx$ converges. If it converges, find its value.

(b) (4 points) Determine whether the integral $\int_1^\infty \frac{1}{x+\sqrt{x}}\,dx$ converges. If it converges, find its value.

5. (a) (4 points) Solve the initial value problem

$$\begin{aligned} y' &= k(P-y)^2 \\ y(0) &= 0. \end{aligned}$$

(b) (2 points) Evaluate $\lim_{x\to\infty} y(x)$.

6. (6 points) The region enclosed by the curves $y = x$ and $y = \sqrt{2x}$ is rotated about the x-axis. Find the volume of the resulting solid.

7. (6 points) Let f be a function such that f'' is continuous, $f(2) = 1$, $f(5) = 8$, $f'(2) = -3$ and $f'(5) = 4$. Compute the value of $\displaystyle\int_2^5 x\, f''(x)\, dx$.

8. (6 points) Determine the radius of convergence of the power series $\displaystyle\sum_{k=1}^{\infty} \frac{(k-1)!}{(k+1)!} x^k$

9. (10 points) Suppose that $P_4(x) = x + 16x^2 - 2x^4$ is the 4^{th}-degree Taylor polynomial approximation for a function $f(x)$ near $x = 0$. Find

(a) $f(0)$

(b) $f'(0)$

(c) $f''(0)$

(d) $f'''(0)$

(e) $f^{(4)}(0)$