



University of California, San Diego
Department of Mathematics

Instructions

1. Write your *Name*, *PID*, *Section*, and *Exam Version* on the front of your Blue Book.
2. No calculators or other electronic devices are allowed during this exam.
3. You may use one page of notes, but no books or other assistance during this exam.
4. Read each question carefully, and answer each question completely.
5. Write your solutions clearly in your Blue Book.
 - (a) Carefully indicate the number and letter of each question and question part.
 - (b) Present your answers in the same order they appear in the exam.
 - (c) Start each problem on a new page.
6. Show all of your work. No credit will be given for unsupported answers, even if correct.
7. Turn in your exam paper with your Blue Book.

0. (1 point) Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

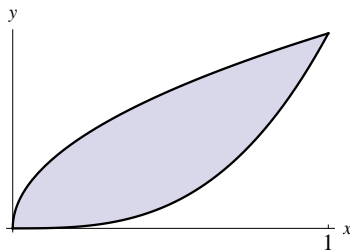
1. (10 points) Compute the definite integral:

$$\int_0^1 \frac{x e^{\sqrt{x^2+1}}}{\sqrt{x^2+1}} dx$$

2. (10 points) Compute the indefinite integral:

$$\int x^2 \sin(3x) dx$$

3. (10 points) The shaded region below is bounded by $y = x^2$ and $y = \sqrt[3]{x}$. Calculate the volume of the solid formed by rotating the region about the y -axis.



4. (10 points) Compute the following indefinite integral using complex exponentials:

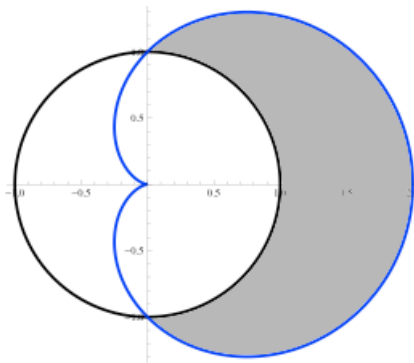
$$\int e^{2x} \cos(x) \sin(x).$$

You need not simplify the result and may leave it in complex exponential form.

5. (10 points) Use the method of partial fractions to compute the indefinite integral:

$$\int \frac{x}{(x-1)(x^2+1)} dx$$

6. (10 points) The two polar curves below are the circle $r = 1$ and the cardioid $r = 1 + \cos(\theta)$. Calculate the area of the shaded region.



7. (10 points) Compute the following definite integral:

$$\int_0^3 \frac{x^2}{\sqrt{36-x^2}} dx$$

8. (a) (5 points) Determine if the following series converges or diverges. State which test you used and make sure to verify all conditions required by that test. **You do not need to compute the value of the series.**

$$\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^2}\right)$$

- (b) (5 points) Explain why you can use (a) to determine whether or not the following improper integral converges or diverges?

$$\int_1^{\infty} \sin\left(\frac{1}{x^2}\right) dx.$$

9. (9 points) Determine if the following series converges absolutely, conditionally, or diverges. State which test you used and make sure to verify all conditions required by that test. **You do not need to compute the value of the series.**

$$\sum_{n=4}^{\infty} (-1)^n \frac{\ln n}{n}$$

10. (10 points) Find the interval of convergence for the following power series:

$$\sum_{n=1}^{\infty} \frac{2n}{3^n} (x-5)^n$$

(This exam is worth 100 points.)