University of California, San Diego
Department of Mathematics

Instructions

1. **Write your *Name*, *PID*, *Section*, and *Exam Version* on the front of your Blue Book.**
2. **No calculators or other electronic devices are allowed during this exam.**
3. **You may use one page of notes, but no books or other assistance during this exam.**
4. **Read each question carefully, and answer each question completely.**
5. **Write your solutions clearly in your Blue Book.**
   (a) **Carefully indicate the number and letter of each question and question part.**
   (b) **Present your answers in the same order they appear in the exam.**
   (c) **Start each problem on a new page.**
6. **Show all of your work. No credit will be given for unsupported answers, even if correct.**
7. **Turn in your exam paper with your Blue Book.**

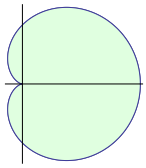
DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO

Question Zero:

0. Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam. [1]

(This exam is worth 70 points.)

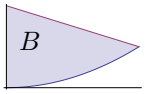
1. Find the area of the cardioid $r = 3 + 3\cos\theta$. [7]



2. Compute the value of the improper integral or show that it does not converge: [8]

$$\int_2^\infty \frac{\ln(7x)}{x^2}\,dx$$

3. Let B be the shaded region bounded by $y = x^2$, $y = 2 - x$, and the y-axis. Find the volume of the solid whose base is B and whose cross-sections perpendicular to the y-axis are squares. [8]



4. Compute the definite integral: [8]

$$\int \frac{x^3 + 2}{x^2(x-1)}\,dx$$

5. Show that the following series converges or diverges: [7]

$$\sum_{n=2}^{\infty} \frac{n!}{n^n}$$

Hint: What is the value of $\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n$?

6. Write the following complex number in polar form: [7]

$$\frac{(2+2i)^4}{(1+\sqrt{3}i)^3}$$

7. Determine if the following series converges absolutely, conditionally, or not at all. (Justify your answer.) [8]

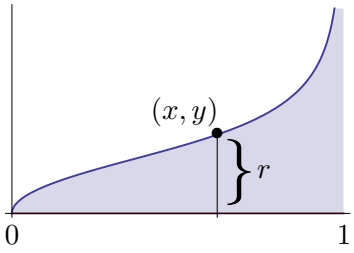
$$\sum_{n=3}^{\infty} \frac{(-1)^n}{(\ln n)^2}$$

8. Find the radius of convergence and the interval of convergence for the following power series: [8]

$$\sum_{n=2}^{\infty} \frac{(x-7)^n}{n \cdot 10^n}$$

9. The region in the first quadrant between $x = 0$ and $x = 1$ and below the graph of $y = \frac{\sqrt{\arcsin(x)}}{(1-x^2)^{1/4}}$ is rotated about the x-axis to obtain a solid of revolution. Compute the volume of this region. [8]

$$V = \int_0^1 \pi r^2\,dx = ?$$



$$y = \frac{\sqrt{\arcsin(x)}}{(1-x^2)^{1/4}}$$