University of California, San Diego
Department of Mathematics

Instructions:

1. Write your *Name*, *PID*, and *Section* on the front of your Blue Book.

2. Write the *Version* of your exam on the front of your Blue Book.

3. No calculators or other electronic devices are allowed during this exam.

4. You may use one page of notes, but no books or other assistance during this exam.

5. Read each question carefully, and answer each question completely.

6. Write your solutions clearly in your Blue Book

   (a) Carefully indicate the number and letter of each question.

   (b) Present your answers in the same order they appear in the exam.

   (c) Start each question on a new page.

7. Show all of your work; no credit will be given for unsupported answers.

1. Let $f(x,y) = x^2 + y^2 + 3x + 2y$.   (16)

   (a) Find the gradient of f at the point $(-1, 1)$.

   (b) Find the directional derivative of f at $(-1, 1)$ in the direction of the vector $\boldsymbol{v} = \langle 4, 3 \rangle$.

   (c) Find all points for which the maximum rate of increase of f is in the same direction as $\langle 1, 1 \rangle$.

2. A surface is given by the formula $xe^y + \dfrac{z}{x} + z\cos(y) = 3$. Find the equation of the tangent plane at the   (6)
point $(1, 0, 1)$. Write your answer as a scalar equation; that is, in the form $Ax + By + Cz = D$.

3. Find the relationship between A and B if $u(x,y) = e^{Ax}\sin(By)$ satisfies the equation $u_x = u_{yy}$.   (6)

4. Consider the function $f(x,y) = x^3 + y^2 - 6x$.   (16)

   (a) Find and classify the critical points of f.

   (b) Find the absolute maximum and minimum values of f on the set shown below, which is bounded
       by the curve $y^2 = -x^3 + 6x$. (*Hint:* Note that $f(x,y) = 0$ on the boundary.)
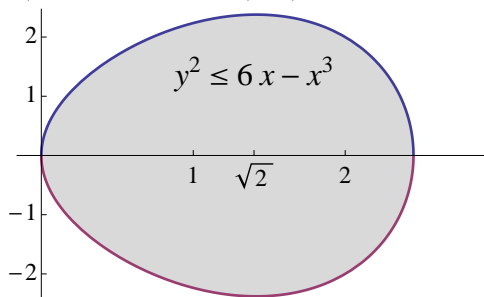
5. Suppose $w = \dfrac{x}{y}e^{z^2}$, where x, y, and z are functions of t. Suppose $x(0) = 2$, $y(0) = 1$, and $z(0) = 0$. If   (6)
you know that $x'(0) = \frac{1}{2}, y'(0) = \frac{1}{3}, z'(0) = \frac{1}{5}$, then what is $\dfrac{dw}{dt}$ at $t = 0$?

(This exam is worth 50 points.)