University of California, San Diego
Department of Mathematics

Name: _____ PID: _____

Discussion Section: _____

Instructions

1. Write your *Name*, *PID*, *Section*, and *Exam Version* on the front of your Blue Book.
2. No calculators or other electronic devices are allowed during this exam.
3. You may use one page of notes, but no books or other assistance during this exam.
4. Read each question carefully, and answer each question completely.
5. Write your solutions clearly in your Blue Book.
   (a) Carefully indicate the number and letter of each question and question part.
   (b) Present your answers in the same order they appear in the exam.
   (c) Start each problem on a new page.
6. Show all of your work. No credit will be given for unsupported answers, even if correct.
7. Turn in your exam paper with your Blue Book.

DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO

Question Zero:

0. Carefully read and complete the instructions at the top of this exam sheet and any additional (1)
   instructions written on the chalkboard during the exam.

(This exam is worth 80 points.)

1. Let $\vec{v} = 4\vec{i} + 3\vec{j} - 2\vec{k}$ and $\vec{w} = 2\vec{i} + \vec{j} - \vec{k}$. $\hfill (9)$

   (a) Compute the angle between $\vec{v}$ and $\vec{w}$.

   (b) Find a unit vector that is orthogonal to both $\vec{v}$ and $\vec{w}$.

   (c) If $\vec{u}$ is a unit vector such that $\vec{u} \cdot \vec{w} = 2$, compute $\|\vec{u} - \vec{w}\|$.

2. Consider the path $\vec{c}(t) = \langle \cos(2t), 9, e^{-2t} \rangle$. $\hfill (10)$

   (a) Find the equation for the tangent line to the curve at $t = 0$.

   (b) Set up an integral that gives the length of the curve traced out by the path $\vec{c}(t)$ between $t = a$ and $t = b$. (Do not evaluate the integral.)

3. Let $f(x, y, z) = xy + \dfrac{x^2}{y} - e^{z^3 - 1}$ and let P be the point $(2, 2, 1)$. $\hfill (10)$

   (a) Find the maximum rate of change of f at P.

   (b) Find the rate of change of f at P in the direction of the origin.

4. Find the equation of the tangent plane to the surface $xy + z^2 = 7$ at the point $(-2, 1, 3)$. $\hfill (5)$

5. Let $\vec{c}(t) = \langle 5t^2, \sqrt{t}, t + \ln t \rangle$ be a path in three dimensional space and suppose that $\vec{r}(t)$ is $\hfill (5)$
   another path such that $\vec{r}(1) = \langle 2, 4, 3 \rangle$ and $\vec{r}'(1) = \langle 1, -2, -1 \rangle$. Compute

$$\frac{d}{dt}\left(\vec{c}(t) \cdot \vec{r}(t) \right)\Big|_{t=1}.$$

6. Let $f(x, y) = x^3 + y^3 - 12x - 3y + 15$. Find the critical points and classify each one as a local $\hfill (10)$
   maximum, local minimum, or saddle point.

7. Let $f(x, y) = x^4 + y^4$. Find all critical points of f subject to the constraint $x^2 + y^2 = 4$ and $\hfill (10)$
   compute the absolute maximum and absolute minimum values of f on the circle of radius 2
   centered at the origin.

8. Integrate the function $f(x, y) = 2xy + 1$ over the region $R = \{(x, y) : x \geq 0, y \geq 0, x + y \leq 1\}$. $\hfill (10)$

9. Evaluate the double integral $\displaystyle\iint_R 3y^2 \, dA$, where R is the region in the first quadrant of the $\hfill (10)$
   xy-plane bounded by the curves $y = x^2$ and $y = x^{1/3}$.