University of California, San Diego
Department of Mathematics

Instructions:

1. **Write your *Name*, *PID*, and *Section* on the front of your Blue Book.**
2. **Write the *Version* of your exam on the front of your Blue Book.**
3. **No calculators or other electronic devices are allowed during this exam.**
4. **You may use one page of notes, but no books or other assistance during this exam.**
5. **Read each question carefully, and answer each question completely.**
6. **Write your solutions clearly in your Blue Book**
   - **(a) Carefully indicate the number and letter of each question.**
   - **(b) Present your answers in the same order they appear in the exam.**
   - **(c) Start each question on a new page.**
7. **Show all of your work; no credit will be given for unsupported answers.**

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1. A triangle has vertices at the points $P = (1, 2, 3)$, $Q = (2, 4, 5)$, and $R = (-1, 3, 5)$. (15)
   - (a) Find parametric equations for the line through the points P and Q. (Your answer should be in the form $x = x(t)$, $y = y(t)$, $z = z(t)$.)
   - (b) Find the equation of the plane containing the triangle. Write your answer as $Ax + By + Cz = D$.
   - (c) The angle at vertex P is called θ. Find $\cos(\theta)$. (Your answer should be a fraction.)

2. A particle moves through space with velocity $\vec{v}(t) = \sqrt{3}\vec{\imath} + \cos(t)\vec{\jmath} - \sin(t)\vec{k}$ (where t is time). (15)
   - (a) Find the length of the path taken by the particle between $t = 1$ and $t = 2$.
   - (b) If the particle's initial position (at time $t = 0$) was $\langle 1, 0, 2\rangle$, then find the position function $\vec{r}(t)$.
   - (c) Find the vector tangent to the path travelled by the particle at $t = 0$.

3. Let $f(x, y) = x^3 + \dfrac{3}{2}y^2 - 12x - 3y + 5$. (10)
   - (a) Find the critical points for f.
   - (b) Classify each critical point as a local maximum, local minimum, or saddle point.

4. Let $g(x, y, z) = \dfrac{y^2}{x} - xz^3$ and let $P = (2, 4, 1)$. (15)
   - (a) Find the gradient of g at the point P.
   - (b) Find the directional derivative if g at the point P in the direction pointing towards the origin.
   - (c) Find the equation of the tangent plane to the surface $g(x, y, z) = 0$ at the point P. Write your answer in the form $Ax + By + Cz = D$

5. Minimize $f(x, y) = x^2 + y^2 + xy + 1$ on the set $xy + 1 \geq 0$. (10)

(Please turn over.)

6. Calculate the iterated integral: $\displaystyle\int_0^1 \int_0^{\sqrt{x}} 4ye^{x^2}\, dy\, dx.$ (10)

7. Change the order of integration: $\displaystyle\int_0^{\pi} \int_y^{\pi} \frac{\sin(x)}{x}\, dx\, dy.$ [**Do not evaluate the integral.**] (5)

8. Evaluate $\displaystyle\iint_D \frac{\ln\left(\sqrt{x^2+y^2}\right)}{x^2+y^2}\, dA$, where D is the region between the circles $x^2+y^2=1$ and $x^2+y^2=4$ where $x \geq 0$. (10)

9. **Set up** a **triple** iterated integral that would give the volume of the solid under the surface $z = e^x \sin^2 y$ that lies above the triangle with vertices $(0,0), (1,1)$ and $(2,0)$. [**Do not evaluate the integral.**] (10)

(This exam is worth 100 points.)