Name: _____ PID: _____

- Print your *NAME* on every page and write your *PID* in the space provided above.
- Show all of your work in the spaces provided. No credit will be given for unsupported answers, even if correct.
- Supporting work for a problem must be on the page containing that problem. No scratch paper will be accepted.
- No calculators, tablets, phones, or other electronic devices are allowed during this exam. You may use one page of handwritten notes, but no books or other assistance.

Do not turn over this page until instructed to do so.

(1 pt) 0. **(Question Zero)** Follow the instructions on this exam and any additional instructions given during the exam.

(6 pt) 1. Let $f(x, y, z) = x^2 + e^{2y} + \cos(3z)$.

(a) Compute the gradient of f at the point $(2, 1, 0)$.

(b) What is the maximal rate of change in f at the point $(2, 1, 0)$?

(6 pt) 2. Let $f(x, y, z) = xe^{yz} - 2x^3 \ln(z)$. Find the rate of change in f at the point $(1, 0, 1)$ in the direction normal to the surface $x^2z + xy^2 + 2yz^2 = 1$ at the point $(1, 0, 1)$.

(6 pt) 3. In polar coordinates, Laplace's Equation has the form

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}=0.$$

Find the value (or values) for the constant a for which $u = 3r^{-2}\cos(a\theta)$ is a solution to this differential equation, or show that there is no value for which it is a solution.

(7 pt) 4. Suppose that $f : \mathbb{R}^2 \to \mathbb{R}^2$ is given by $f(x, y) = (x^2 + y, xy^2)$, and suppose the path $\mathbf{c} : \mathbb{R} \to \mathbb{R}^2$ is such that $\mathbf{c}(0) = (1, 2)$ and $\mathbf{c}'(0) = (3, 4)$. Find the tangent vector to the path $f \circ \mathbf{c}$ at $t = 0$.

(9 pt) 5. Let $f(x, y) = -x^4 - 2y^2 + 4xy$.

(a) Find the critical points of f

(b) Use the second derivatives test to classify the critical points of f, of show it is inconclusive.