Name: ______________________________ PID: ______________

- Print your *NAME* on every page and write your *PID* in the space provided above.
- Show all of your work in the spaces provided. No credit will be given for unsupported answers, even if correct.
- Supporting work for a problem must be on the page containing that problem. No scratch paper will be accepted.
- No calculators, tablets, phones, or other electronic devices are allowed during this exam. You may use one page of handwritten notes, but no books or other assistance.
- Sit in your assigned seat.

Do not turn over this page until instructed to do so.

(1 pt) 0. **(Question Zero)** Follow the instructions on this exam and any additional instructions given during the exam.

(4 pt) 1. The motion of a particle in space at time t is given by the position function

$$\mathbf{c}(t) = 3\cos(t)\mathbf{i} + 3\sin(t)\mathbf{j} + t\mathbf{k},$$

where $t \geq 0$. Compute the velocity and the speed of the particle at time t.

(8 pt) 2. Let $f(x, y) = x^2y^2 - x$.

(a) Compute the gradient of f at the point $(2, 1)$.

(b) Write the equation for the tangent plane at $(2, 1, f(2, 1))$.

(c) Use linear approximation to approximate the value of $f(1.9, 1.2)$.

(d) Compute the rate of change in f at $(2, 1)$ in the direction of $-\mathbf{i} + \mathbf{j}$.

(6 pt) 3. Let $\mathbf{r}(t) = (e^t, \sin(t), t^2)$ and suppose $\mathbf{c}(t)$ is a path having tangent vector $(-3, 2, 1)$ at the point $(1, 0, 2)$ when $t = 3$. Compute

$$\frac{d}{dt}\Big(\mathbf{c}(t) \cdot \mathbf{r}(t)\Big)\Big|_{t=3}$$

(6 pt) 4. Suppose that a particle following the path $\mathbf{c}(t) = (\sin(e^t), t, 4 - t^3)$ flies off on a tangent at $t = 1$. Compute the position of the particle at the time $t = 2$.

(6 pt) 5. Let $F(x, y, z)$ be a C^2 function of three variables for which $\nabla F(1, -1, \sqrt{2}) = (1, 2, -2)$. Let

$$x = \rho \sin(\phi) \cos(\theta), \quad y = \rho \sin(\phi) \sin(\theta), \quad z = \rho \cos(\phi).$$

Use the Chain Rule to evaluate $\frac{\partial F}{\partial \phi}$ at $(\rho, \phi, \theta) = (2, \frac{\pi}{4}, -\frac{\pi}{4})$.

(6 pt) 6. Evaluate $\displaystyle\int_0^{1/4}\int_{\sqrt{y}}^{1/2} \frac{e^x}{x}\,dx\,dy$.

(6 pt) 7. Let

$$\iint_R f\,dA = \int_0^2 \int_{x^2}^{\sqrt{8x}} f(x,y)\,dy\,dx,$$

where f is a continuous function on the closed and bounded region R in the xy-plane. Sketch the region R and use it to rewrite the double integral $\iint_R f\,dA$ as an iterated integral where the order of integration has been changed from what is given above.

(6 pt) 8. Evaluate the double integral of $f(x, y) = 1 - x$ over the triangle R having vertices at $(0, 0)$, $(1, 1)$, and $(-2, 1)$.

(6 pt) 9. Let $f(x,y) = x^2 + y^2 - 2x - 4y$. Find the absolute maximum and absolute minimum values of f on the region defined by the inequalities $x^2 + y^2 \leq 2$ and $y \geq 0$.