University of California, San Diego
Department of Mathematics

Name: _____ PID: _____

Date: _____ Discussion Section: _____

1. **Write your *Name*, *PID*, and *Discussion Section* in the spaces provided above.**
2. **Make sure your *Name* is on every page.**
3. **No calculators, tablets, phones, or other electronic devices are allowed during this exam.**
4. **Put away ANY devices that can be used for communication or can access the Internet.**
5. **You may use one page of handwritten notes (both sides), but no books or other assistance during this exam.**
6. **Write your solutions clearly in the spaces provided.**
7. **Show all of your work; no credit will be given for unsupported answers.**

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Question Zero:

(1 points) 0. Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

(This exam is worth 30 points.)

(6 points) 1. (a) Find an explicit solution to the initial value problem:

$$\frac{dy}{dt} = \frac{1}{y} - y, \quad y(0) = 3$$

(2 points) (b) Find all equilibrium solutions to the differential equation $\dfrac{dy}{dt} = \dfrac{1}{y} - y$.

(3 points) 2. (a) Find the value of the constant b for which the given equation is exact:

$$\left(3ye^{2xy} + 2x\right) + \left(bxe^{2xy} + y^3\right)\frac{dy}{dx} = 0$$

(6 points) (b) Use the integrating factor $\mu(x, y) = y$ to find an *implicit* solution to the DE:

$$y + \left(2x + e^{y^2}\right) \frac{dy}{dx} = 0$$

(6 points) 3. Find the general solution to the differential equation:

$$y'' + 2y' + 2y = 0$$

(6 points) 4. The functions $y_1 = t^2$ and $y_2 = \frac{1}{t}$ are solutions to the differential equation $t^2y'' - 2y = 0$ on the interval $t > 0$. Show that y_1 and y_2 form a fundamental set of solutions and solve the initial value problem

$$t^2y'' - 2y = 0, \quad y(1) = 2, \quad y'(1) = 0$$