Name: _____ PID: _____

1. **Write your *NAME* on every page and your *PID* in the space provided above.**
2. **No calculators, tablets, phones, or other electronic devices are allowed during this exam.**
3. **You may use one page of handwritten notes, but no books or other assistance during this exam.**
4. **Write your solutions clearly in the spaces provided.**
5. **Show all of your work; no credit will be given for unsupported answers.**

$f(t)$	$\mathcal{L}\{f(t)\}$	
1	$\frac{1}{s}$	$s > 0$
e^{at}	$\frac{1}{s-a}$	$s > a$
t^n	$\frac{n!}{s^{n+1}}$	$s > 0$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$s > 0$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$s > 0$
$\sinh(at)$	$\frac{a}{s^2-a^2}$	$s > \lvert a\rvert$
$\cosh(at)$	$\frac{s}{s^2-a^2}$	$s > \lvert a\rvert$
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	$s > a$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$	$s > a$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$	$s > a$
$u_c(t)$	$\frac{e^{-cs}}{s}$	$s > 0$
$u_c(t)f(t-c)$	$e^{cs}F(s)$	
$e^{ct}f(t)$	$F(s-c)$	

(1 points) 0. **(Question Zero)** Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

(6 points) 1. Solve the initial value problem. Assume $t > 0$. Your answer should be an *explicit* function of t.

$$t^3y' + 4t^2y = e^{t^2}, \quad y(1) = e$$

(6 points) 2. Solve the initial value problem. Your answer should be an *explicit* function of x.

$$y' = xy^2e^x, \quad y(0) = 3$$

(6 points) 3. Find the general solution of the second order differential equation.

$$y'' + 3y' + 6y = 2t$$

(6 points) 4. Find the solution to the initial value problem:

$$\mathbf{x}' = \begin{pmatrix} 3 & 5 \\ 0 & -1 \end{pmatrix} \mathbf{x}, \qquad \mathbf{x}(0) = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$$

(6 points) 5. Find the 2×2 linear differential system that has the following fundamental set of solutions on the interval $(0, \infty)$:

$$\mathbf{x}^{(1)} = \begin{pmatrix} t^2 \\ 3t \end{pmatrix} \quad \text{and} \quad \mathbf{x}^{(2)} = \begin{pmatrix} e^{2t} \\ 0 \end{pmatrix}$$

(6 points) 6. Solve the following initial value problem:

$$y^{(4)} - 4y = 0; \qquad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 0, \quad y'''(0) = -2$$

(6 points) 7. Find the Laplace transform of the function

$$f(t) = \begin{cases} 0 & \text{if } t < 2 \\ (t-2)^2 & \text{if } 2 \le t < 5 \\ 4 & \text{if } t \ge 5 \end{cases}$$

(7 points) 8. Find the general solution for the differential equation, given that y_1 is known to be a solution:

$$t^2y'' - 2y = 0, \qquad y_1 = t^2, \qquad t > 0.$$