Name: ______________________________ PID: ______________

- Print your *NAME* on every page and write your *PID* in the space provided above.
- Show all of your work in the spaces provided. No credit will be given for unsupported answers, even if correct.
- Supporting work for a problem must be on the page containing that problem. No scratch paper will be accepted.
- No calculators, tablets, phones, or other electronic devices are allowed during this exam. You may use one page of handwritten notes, but no books or other assistance.
- Sit in your assigned seat.

$f(t)$	$\mathcal{L}\{f(t)\}$		$f(t)$	$\mathcal{L}\{f(t)\}$	
1	$\frac{1}{s}$	$s>0$	$\sinh(at)$	$\frac{a}{s^2-a^2}$	$s>\lvert a\rvert$
e^{at}	$\frac{1}{s-a}$	$s>a$	$\cosh(at)$	$\frac{s}{s^2-a^2}$	$s>\lvert a\rvert$
t^n	$\frac{n!}{s^{n+1}}$	$s>0$	$u(t-c)$	$\frac{e^{-cs}}{s}$	$s>0$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$s>0$	$u(t-c)f(t-c)$	$e^{-cs}F(s)$	
$\cos(at)$	$\frac{s}{s^2+a^2}$	$s>0$	$e^{ct}f(t)$	$F(s-c)$	
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	$s>a$	$\delta(t-c)$	e^{-cs}	
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$	$s>a$	$f*g$	$F(s)G(s)$	
t^ne^{at}	$\frac{n!}{(s-a)^{n+1}}$	$s>a$	$f^{(n)}(t)$	$s^nF(s)-\sum s^{n-k}f^{(k-1)}(0)$	

Hint: $$\frac{1}{s^2(s^2+a^2)}=\frac{1/a^2}{s^2}-\frac{1/a^2}{s^2+a^2}$$

Do not turn over this page until instructed to do so.

(1 pt) 0. **(Question Zero)** Follow the instructions on this exam and any additional instructions given during the exam.

(5 pt) 1. Suppose that $y = \phi(t)$ is a solution to the initial value problem

$$\frac{dy}{dt} = e^{3t}(y^2 - 4), \quad y(0) = -3.$$

(a) List the equilibrium solutions of the differential equation. (b) Compute $\lim_{t \to \infty} \phi(t)$.

(6 pt) 2. Suppose the following differential equation is exact:

$$\left(x^2y^3 - 2xy\right)dx + \left(x^3y^2 + g(x) + \frac{1}{1+y^2}\right)dy = 0.$$

Find $g(x)$ and solve the differential equation. (Leave your answer in implicit form.)

(6 pt) 3. Find the general solution of the second order differential equation: $y'' + 3y' = 4t$

(6 pt) 4. Find the general solution: $\mathbf{x}' = \begin{pmatrix} 1 & 3 \\ -3 & -5 \end{pmatrix} \mathbf{x}$

(6 pt) 5. Solve the initial value problem using a power series centered at $x = 0$. Write out the first *three* nonzero terms of the infinite series: $y'' + xy = 0, \quad y(0) = 3, \quad y'(0) = 0.$

(6 pt) 6. Compute $\mathcal{L}^{-1}\left\{\dfrac{2s}{s^2+4s+13}\right\}$.

(7 pt) 7. Solve the following initial value problem. (Use the hint on the cover page.)

$$y'' + 4y = \begin{cases} 0 & \text{if } t < 3 \\ t-3 & \text{if } t \geq 3 \end{cases}; \qquad y(0) = 1, \quad y'(0) = 1.$$

(7 pt) 8. Find the general solution for the differential equation, given that y_1 is known to be a solution:

$$ty'' - (1+t)y' + y = 0, \qquad y_1 = e^t, \qquad t > 0.$$

You may use this page for scratch paper, but nothing written on this page may be used as supporting work.