Name: ________________________ **PID:** ______________

TA: ____________________ **Sec. No:** _____ **Sec. Time:** _____

Math 20D.
Midterm Exam 2
May 22, 2009

Turn off and put away your cell phone.
No calculators or any other electronic devices are allowed during this exam.
You may use one page of notes, but no books or other assistance during this exam.
Read each question carefully, and answer each question completely.
Show all of your work; no credit will be given for unsupported answers.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
If any question is not clear, ask for clarification.

#	Points	Score
1	8	
2	8	
3	10	
Σ	26	

1. (8 points) Consider the system of two first order initial value problems

$$\begin{cases} x_1' = 2x_1 + 4x_2, & x_1(0) = 4 \\ x_2' = x_1 - x_2, & x_2(0) = 1 \end{cases}$$

(a) Write the system of initial value problems as a matrix initial value problem $\mathbf{x}' = \mathbf{A}\mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix}$.

(b) Solve the matrix initial value problem. Be sure your solution $\mathbf{x}(t)$ satisfies the initial condition $\mathbf{x}(0) = \begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix}$.

(c) Describe the behavior of the solution to the matrix initial value problem in the phase plane as $t \to \infty$.

2. (8 points) Consider the homogeneous system

$$\mathbf{x}' = \begin{pmatrix} -6 & 4 \\ -1 & -2 \end{pmatrix} \mathbf{x}.$$

The characteristic equation $|\mathbf{A} - r\mathbf{I}| = 0$ simplifies to $(r+4)^2 = 0$. So, the coefficient matrix has one repeated eigenvalue $r = -4$, and $\mathbf{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ is a corresponding eigenvector. Therefore, $\mathbf{x}^{(1)}(t) = \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-4t}$ is a solution.

Find a second solution $\mathbf{x}^{(2)}(t)$ that is linearly independent of $\mathbf{x}^{(1)}(t)$.

3. (10 points) Consider the nonhomogeneous system

$$\mathbf{x}' = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 3 \\ 1 \end{pmatrix} e^t.$$

The matrix $\begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix}$ has eigenvalues $r_1 = i$ and $r_2 = -i$, with corresponding eigenvectors $\mathbf{v_1} = \begin{pmatrix} 2+i \\ 1 \end{pmatrix}$ and $\mathbf{v_2} = \begin{pmatrix} 2-i \\ 1 \end{pmatrix}$, respectively.

(a) Use undetermined coefficients to find a particular solution $\mathbf{y}(t)$ to the nonhomogeneous system.

(b) Write down the general solution to the nonhomogeneous system using only real-valued functions.