Name: ______________________ **PID:** ______________

TA: ________________ **Sec. No:** ____ **Sec. Time:** ____

Math 20D.
Final Examination
December 9, 2008

Turn off and put away your cell phone.
No calculators or any other electronic devices are allowed during this exam.
You may use one page of notes, but no books or other assistance during this exam.
Read each question carefully, and answer each question completely.
Show all of your work; no credit will be given for unsupported answers.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
If any question is not clear, ask for clarification.

#	Points	Score
1	10	
2	10	
3	10	
4	8	
5	8	
6	10	
Σ	56	

1. Consider the following differential equation:

$$y'' - xy' - y = 0.$$

   (a) (6 points) Find the recurrence relation for the power series solution about the point $x_0 = 0$.

   (b) (4 points) Write the first four terms of each of the two linearly independent power series solutions.

2. (10 points) Use the Laplace transform to solve the following initial value problem:

$$\begin{cases} y'' + 9y = \delta(t-3) \\ y(0) = 0, \quad y'(0) = 3 \end{cases}$$

Refer to the attached table of Laplace transforms.

3. Consider the following homogeneous linear system of differential equations:

$$\mathbf{x}' = \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix} \mathbf{x}.$$

(a) (4 points) Find the eigenvalues and for each eigenvalue, find an associated eigenvector.

(b) (6 points) Find two linearly independent *real-valued* solutions to the linear system of differential equations.

4. Consider the differential equation

$$-2xy\sin(x^2y) + [2y - x^2\sin(x^2y)]y' = 0.$$

(a) (2 points) Verify that the differential equation is exact.

(b) (6 points) Solve the differential equation. Leave the result in implicit form: do not try to solve for $y(x)$ explicitly.

5. (8 points) Solve the linear first-order initial value problem

$$\begin{cases} t^2 y' = 3ty - 3t - 1 \\ y(0) = 1 \end{cases}$$

6. Consider the following homogeneous linear system of differential equations:

$$\mathbf{x}' = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} \mathbf{x}$$

(a) (4 points) The coefficient matrix has a repeated eigenvalue. Find it and an associated eigenvector; then, write down the corresponding solution.

(b) (6 points) Find another solution that is linearly independent from the solution you found in part (a).

Table of Elementary Laplace Transforms

	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1.	1	$\frac{1}{s}, \quad s > 0$
2.	e^{at}	$\frac{1}{s-a}, \quad s > a$
3.	$t^n, \quad n = \text{positive integer}$	$\frac{n!}{s^{n+1}}, \quad s > 0$
4.	$t^p, \quad p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$
5.	$\sin(at)$	$\frac{a}{s^2+a^2}, \quad s > 0$
6.	$\cos(at)$	$\frac{s}{s^2+a^2}, \quad s > 0$
7.	$\sinh(at)$	$\frac{a}{s^2-a^2}, \quad s > \lvert a \rvert$
8.	$\cosh(at)$	$\frac{s}{s^2-a^2}, \quad s > \lvert a \rvert$
9.	$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}, \quad s > a$
10.	$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}, \quad s > a$
11.	$t^n e^{at}, \quad n = \text{positive integer}$	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$
12.	$u_c(t)$	$\frac{e^{-cs}}{s}, \quad s > 0$
13.	$u_c(t)f(t-c)$	$e^{-cs}F(s)$
14.	$e^{ct}f(t)$	$F(s-c)$
15.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$
16.	$\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$
17.	$\delta(t-c)$	e^{-cs}
18.	$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$
19.	$(-t)^nf(t)$	$F^{(n)}(s)$