HOMEWORK 4
DUE 29 APRIL 2016

Part I.

1. Show that $R_1 \otimes \mathbb{Z} R_2$ is the coproduct of $R_1$ and $R_2$ in the category of commutative rings.

2. (a) Let $A$ be an $R$-algebra and $I$ an ideal in $R$. Show that $R/I \otimes_R A \simeq A/J$ as $R$-algebras, where $J = I^e$ is the extension of the ideal $I$ to an ideal of $A$ (i.e., the ideal of $A$ generated by the image of $I$ via the structure homomorphism).
(b) If $A$ is an $R$-algebra and $I$ an ideal of $R[X]$, show that $A \otimes_R (R[X]/I) \simeq A[X]/J$, where $J$ is the ideal of $A[X]$ generated by the image of $I$, i.e., the extension of $I$ to $A[X]$ via the map $R[X] \longrightarrow A[X]$ induced by the structure homomorphism of $A$.

3. Show that
   (a) $R[X] \otimes_R R[Y] \simeq R[X, Y]$ as $R$-algebras.
   (b) $R/I \otimes_R R/J \simeq R/(I + J)$ for any two ideals $I, J$ of $R$.

Part II. From Atiyah-MacDonald

Chapter 2: 3, 4, 6, 17, 20

Bonus.

B1. Let
   \[ R = \{ f : [0, 1] \longrightarrow \mathbb{R}; f \text{ is continuous and } f(0) = f(1) \} \]
   and
   \[ M = \{ g : [0, 1] \longrightarrow \mathbb{R}; g \text{ is continuous and } g(0) = -g(1) \} . \]

Then $R$ is a commutative ring under addition and multiplication of functions and $M$ is an $R$-module. Is $M$ free as an $R$-module? Is it projective?

Atiyah & Macdonald, Chapter 2: 7, 21