

## Problems from Lecture 2

1. (Spencer, p. 12) Show that  $R(3, t) = t^{3/2+o(1)}$  using the deletion method. Note how the usual probabilistic method is ineffective at bounding  $R(3, t)$ .
2. Let  $q(n)$  denote the largest number of points that can be placed on an  $n \times n$  lattice such that no four points form a rectangle. Use the deletion method to find a lower bound for  $q(n)$ .
3. (Alon and Spencer, p. 39) Prove that every 3-uniform hypergraph with  $n$  vertices and  $m \geq n/3$  edges contains an independent set (i.e. a set of vertices containing no edges) of size at least

$$\frac{2n^{3/2}}{3\sqrt{3}\sqrt{m}}.$$

4. \*(Alon and Spencer, p. 40) Show that there is a finite  $n_0$  such that any directed graph on  $n > n_0$  vertices in which each outdegree is at least  $\log_2 n - \frac{1}{10} \log_2 \log_2 n$  contains an even simple directed cycle. (A directed cycle  $v_1 \rightarrow \dots \rightarrow v_n \rightarrow v_1$  is simple if  $v_1, \dots, v_n$  are distinct).