Problems inspired by Lecture 6: Order from Chaos

1. The Spencer shows that, for any initial conditions, a $n \times n$ grid of lights with a switch for each row and column can be set to have at least $cn^{3/2}$ more lights on than off, and that this is the best we can do for the worst initial conditions. Generalize this to an $r \times s$ grid.

2. What happens in the 3-dimensional lights problem? For an $n \times n \times n$ grid of lights, there are 2 natural ways to generalize this game — a switch could control a line of lights or a plane of lights, giving either $3n^2$ or $3n$ switches respectively. Play with whichever one sounds better.

3. Let $Sum_n = X_1 + \ldots + X_n$, where each X_k is independent with $P(X_k = 1) = P(X_k = -1) = \frac{1}{2}$. Show that
$$E(|Sum_n|) = n2^{1-n}\binom{n-1}{[(n-1)]/2}$$
Note: This was a Putnam problem, and Spencer says it was hard. But we can do it!