Problems for Lecture 5

1. Complete the proof of lindisc $\mathcal{A} \leq$ herdisc $\mathcal{A}$ in the case where p_i don't have finite binary expansions. (Don't be a critic, a neophyte, or a computer scientist. Use compactness!)

2. Spencer gives an outline of a proof (due to Lovasz) that lindisc $\mathcal{A} \leq \frac{n}{n+1}$ when $\mathcal{A}$ consists of all subintervals of $[n]$. Fill in the details of this proof. Note how, together with Spencer's own reasoning, this completely establishes the exact values of (lin)(her)disc for examples (1) and (3) on pp. 37-38. (Alternatively, you might wish to come up with your own proof for example (1) which establishes the upper bound on lindisc directly).

3. Prove the following statements. (Or disprove them in case I messed up.) If H is an $m \times n$ matrix and each row i satisfies $\sum_{j=1}^{n} |h_{ij}| \leq K$, then lindisc $H \leq K/2$. In particular, if G is a graph, then lindisc $G \leq 1$. (In this context, a graph is a set family consisting of 2-sets, and in fact 1-sets don't hurt either). lindisc $G = 1$ if and only if G is not 2-colorable. There is a sequence of 2-colorable graphs G_n such that lindisc $G_n \to 1$.

4* A random case to think about: Assume $\Omega = [n]$ and $\mathcal{A}$ consists of sets of size $n-1$ or n. How much can we say about the various types of discrepancy in this case? Conceivably we might be able to find the exact values of all four as functions of n, $m = |\mathcal{A}|$, and whether or not $[n] \in \mathcal{A}$. Barring that, try to come up with the best upper and lower bounds that you can.

5* Look up Hoffman's disproof of the Sos conjecture, and present it during the next problem section (possibly in sketch form).