Additional problems for Homework 1

**Problem 1.** Let $X$ be a nonempty set. Let $\mathcal{E} = \{A_k\}_{k \in \mathbb{N}}$ be a collection of disjoint subsets of $X$ such that $\bigcup_{k \in \mathbb{N}} A_k = X$. Prove that the $\sigma$-algebra $\mathcal{M}(\mathcal{E})$ generated by $\mathcal{E}$ is equal to $\{\bigcup_{k \in S} A_k \mid S \subset \mathbb{N} \text{ such that } S \text{ is countable or } S^c \text{ is countable}\}$.

**Problem 2.** Let $X$ be a nonempty finite set. Let $\mathcal{A} \subset \mathcal{P}(X)$ be an algebra. Prove that there exists a collection $\{A_k\}_{k=1}^n$ of disjoint subsets of $X$ such that $\bigcup_{k=1}^n A_k = X$ and $\mathcal{A} = \{\bigcup_{k \in S} A_k \mid S \subset \{1, 2, \ldots, n\}\}$. 